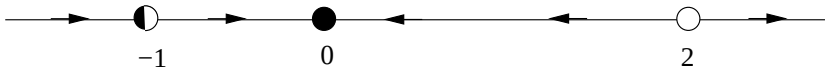
Math 5110/6830
Instructor: Alla Borisyuk
Homework 5.1
Due: October 17

1. Analyze the following equations graphically. In each case sketch the vector field on the real line, find all the fixed points, classify their stability, an sketch the graph of $x(t)$ for different initial conditions. Then try to solve the equation analytically. It may not be possible! - state it if you think so.
a) $\dot{x} = x - x^3$
b) $\dot{x} = e^{-x} \sin x$

2. Suggest an equation that is consistent with the following phase portrait:



3. Consider a bacterial population whose growth rate is

$$dN/dt = K(t)N$$

. Show that

$$N(t) = N_0 \exp\left(\int_0^t K(s)ds\right).$$

4. Solve the logistic equation $\dot{N} = rN(1 - N/K)$ analytically for an arbitrary initial condition N_0. (**Hint:** Separate variables and integrate, using partial fractions OR make the change of variables $x = 1/N$).

5. Cancerous tumor growth can be modeled by the Gompertz law:

$$\dot{N} = -aN \ln(bN),$$

where $N(t)$ is proportional to the number of cells in the tumor and $a, b > 0$ are parameters. Sketch the vector field and then graph $N(T)$ for various initial values.