

Math 5110/6830
Instructor: Alla Borisjuk
Homework 3.2
Due: September 19

1. (dV 2.4.16) Consider the Love Affairs model that we discussed in class in a “feeling preserving” mode when $a_R + p_J = 1$ and $a_J + p_R = 1$.

a) Write down the system of equations for the fixed points (in matrix form, as we did in class). Show that in the “feeling preserving” mode the determinant of this system is equal to zero.

b) Show that the two eigenvalues of the Jacobian matrix are

$$\lambda_1 = 1,$$

$$\lambda_2 = a_R + a_J - 1.$$

2. (dV 2.14.17) Consider the Nicholson-Bailey model that we considered in class.

a) Show that the fixed points of the system are the trivial fixed point

$$(H_1^*, P_1^*) = (0, 0)$$

and the non-trivial fixed point

$$(H_1^*, H_2^*) = \left(\frac{k \ln k}{ac(k-1)}, \frac{\ln k}{a} \right),$$

provided $k > 1$. Why did we need the restriction $k > 1$?

b) Show that the non-trivial fixed point is unstable by checking that the second Jury condition is not satisfied at that point. (Hint: you may want to consider $f(k) = k \ln k - k + 1$ and show $f(k) > f(1) = 0$ for $k > 1$.)

3. One possible modification for the Nicholson-Bailey model is to limit the growth of the host species in isolation by assuming intra-species competition and, subsequently, an existence of the carrying capacity (maximum possible level of the population). Beddington, Free and Lawton realized this in the following model:

$$\begin{aligned} H_{n+1} &= e^{r(1-H_n/K)} H_n e^{-aP_n}, \\ P_{n+1} &= cH_n [1 - e^{-aP_n}]. \end{aligned} \tag{1}$$

a) Determine all fixed points

b) Under what conditions on the model parameters are fixed points stable? Unstable?

c) **(Extra credit)** Can you suggest some other factors that may stabilize the coexistence solution in Nicholson-Bailey model? You can describe the modifications in words or pictures and, perhaps, formulas. You do not need to analyze these new models.

4. For the Poisson process:

a) Sketch $p(i)$ as a function of i .

b) Suggest two examples of physical processes that can be described as a Poisson process.

c) Suppose that we modify the assumptions of the Nicholson-Bailey model in the following way: if a host has one encounter with a parasitoid it gives rise to c parasitoid progeny, but if it has two or more encounters it gives rise to $2c$ parasitoid progeny. Write down a new model incorporating this assumption.