

Math 5110/6830
Instructor: Alla Borisjuk
Homework 3.1
Due: September 19 at 8 pm

1. (dV 2.4.13) Consider the following competition model:

$$\begin{aligned}A_{n+1} &= \mu_1 A_n - \mu_3 A_n B_n, \\B_{n+1} &= \mu_2 B_n - \mu_4 A_n B_n,\end{aligned}\tag{1}$$

where $\mu_1, \mu_2, \mu_3, \mu_4$ are positive.

- a) Find all fixed points.
b) Determine the stability of the fixed points for $\mu_1 = 1.2, \mu_2 = 1.3, \mu_3 = 0.001$, and $\mu_4 = 0.002$.
2. (dV 2.4.14) Consider the following model for the spread of an infectious disease (such as the flu or the common cold) through a population of size N :

$$\begin{aligned}I_{n+1} &= I_n + kI_n(N - I_n), \\I_0 &= 1,\end{aligned}\tag{2}$$

where I_n is the number of infected (and infectious) individuals on day n .

- a) What is k in this model?
b) What does the model predict, if we assume $kN < 2$?

The above model does not include recovery. Consider recovery with immunity (i.e. once a person recovers, (s)he cannot get sick the second time), and assume that an individual recovers exactly in d days.

- c) Modify the model to include the recovery with immunity. Explain (justify) your model. What additional assumptions have you made?
d) What are the steady states of your modified model?
e) Study the stability of the steady states.