

Math 5110/6830
Instructor: Alla Borisjuk
Homework 2.1
Due: September 12 at 8 pm

1. (dV 2.4.5) Consider the survival of a population of whales, and assume that if the number of whales falls below a minimum survival level m , then the species will become extinct. In addition, assume that the population is limited by the carrying capacity M of the environment. That is, if the whale population is above M , then it will experience a decline because the environment cannot sustain that large a population level.

a) Let a_n represent the whale population after n years. Consider the model

$$a_{n+1} = a_n + k(M - a_n)(a_n - m),$$

where $k > 0$. Describe the model: what are the variables, the parameters, what does each term mean. Does this model fit the description above?

b) Find the fixed points of the model, and determine their stability via linearization. Assume that $M = 5000$, $m = 100$, and $k = 0.0001$.

c) Perform a graphical stability analysis, using cobwebbing. Are your results consistent with the results from b)?

d) Sketch the graphs of a_n versus n for various representative initial conditions.

e) This model fails when the initial population size a_0 is too small or too large. Why?

f) (extra credit) Can you suggest (in words or in graphs) how to overcome the problem with too small or too large population?

2. Consider a discrete-time model of the concentration of medication in the bloodstream, M_t ,

$$M_{t+1} = M_t - f(M_t)M_t + S$$

where $f(M_t)$ is the fraction absorbed and S is the daily dosage. Suppose that

$$f(M) = \frac{M^p}{K^p + M^p}.$$

Set $K = 2$ and $S = 1$.

a) Show that $M^* = 2$ is the equilibrium point for any value of p .

b) Study the stability of the equilibrium for different values of p : when is it stable? unstable? when does the solution oscillate toward the equilibrium?

c) Sketch a cobweb diagram of a case when $M^* = 2$ is unstable.