Math 5110/6830
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Homework 10
Due: December 11

1. We considered in class a discrete model for the spread of disease:

$$C_{t+1} = fC_tS_t,$$

$$S_{t+1} = S_t + B - fC_tS_t.$$

a)Perform stability analysis of the steady states using cob-webbing
b)Show by studying the state's stability analytically that a small deviation from steady state can result in oscillatory behavior
c)Using a calculator or a Matlab code show that the time course of disease exhibits waves. Use $B = 12$ births per 1000 people, $f = 0.3 \times 10^{-4}$, $S_0 = 2000$, $C_0 = 20$.

2. **a)**Suppose that N_t and P_t are populations of hosts and parasitoids, resepctively, in year t. Both species live at most 1 year. A fraction $f = \frac{1}{1+P_t^2}$ of hosts survive parasitoid attacks, and each of these survivors has $\frac{\lambda}{1+N_t}$ offsprings that survive until year $t+1$. Each host that does not survive produces exactly c new parasitoids that survive until year $t+1$. Write a discrete time dynamical system describing these two populations.

b)An enzyme E complexes with a substrate S. The resulting complex C can break down into two molecules of the enzyme. All reactions are reversible. Write differential equations describing this system.

c)A species has three types of individuals: Wee babies (age 1), juveniles (age 2) and adults (ages 3 and older). All are female. Wee babies have a 0.5 chance of surviving to the juvenile state, juveniles have 0.5 chance of surving to adulthood and adults have a 0.4 chance of surviving. Adults produce 2 wee babies. Write equations that describe this population. How would you figure out if the population would grow?

3. There are two populations, described by S_t and I_t, respectively. Their dynamics is governed by

$$S_{t+1} = \left(\frac{\rho S_t}{k_1 + S_t}\right)\left(\frac{k_3}{k_3 + I_t}\right),$$

$$I_{t+1} = \frac{\beta S_t}{k_2 + S_t} I_t.$$

a)What would happen to S in the absence of I? Draw a digram to illustrate your answer

b)How do S and I influence each other's reproduction?

c)Find the equilibria of the two-dimensional system. What are the conditions for existence of an equilibrium with $I = 0$ and $S > 0$? What are the conditions for existence of an equilibrium with $I > 0$ and $S > 0$?

d)Assuming that two non-negative equilibria exist, find their stability and sketch a phase-plane.