Math 5110/6830
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Homework 1.1
Due: September 5

1. (EK 1.1) Consider the difference equation

$$x_{n+2} - 3x_{n+1} + 2x_n = 0.$$

a) Show that the general solution to this equation is

$$x_n = A_1 + 2^n A_2.$$

(Hint: when does the corresponding system of equation has a non-trivial solution?)

b) Suppose that $x_0 = 10$ and $x_1 = 20$. The above equation with these initial conditions is called *the initial value problem.* What is the solution?

2. (EK 1.2) Solve the following difference equations subject to the specified x values and sketch the solution (by hand!). Describe the solutions (increases, decreases, oscillations, characteristic time scale)

a) $x_n - 5x_{n-1} + 6x_{n-2} = 0; x_0 = 2, x_1 = 5.$

b) $x_n + x_{n-2} = 0; x_1 = 3, x_2 = 5.$

3. (EK 1.6, 1.7) In each of the following systems determine the eigenvalues. Find the solution of the system and sketch the solution.

a)

$$\begin{aligned} x_{n+1} &= 3x_n + 2y_n \\ y_{n+1} &= x_n + 4y_n \end{aligned} \qquad (1)$$

with $x_0 = 1$, $y_0 = 3$.

b)

$$\begin{aligned} x_{n+1} &= -x_n + 3y_n \\ y_{n+1} &= \frac{y_n}{3} \end{aligned} \qquad (2)$$

with $x_0 = 2$, $y_0 = 3$.

4. (EK 1.14) In 1202 Fibonacci (Leonardo of Pisa) posed and solved the following problem (perhaps, the first mathematical idealization of a biological phenomenon). Suppose that every pair of rabbits can reproduce only twice, when they are one and two months old, and that every time they produce exactly one new pair of rabbits (one male and one female). Assume that all rabbits survive. To solve the problem, let us define:
R_n^0 = number of newborn pairs in generation n,
R_n^1 = number of one-month-old pairs in generation n,
R_n^2 = number of two-month-old pairs in generation n.

a) Show that

$$R^0_{n+1} = R^0_n + R^0_{n-1}.$$

Can you explain in words why this result makes sense.

b) Starting with a single newborn pair in the first generation ($R^0_0 = 1$), how many pairs will be there after n generations?

c) (extra credit) The relation that we derived in a): $x_{n+2} = x_{n+1} + x_n$ with $x_0 = 0$, $x_1 = 1$ generates a sequence of numbers, known as Fibonacci numbers. List a few examples of where these numbers are found (in nature, art, or geometry or elsewhere). You can use anything (yes, google also) to answer this. Why do you think these numbers are so common?