Math 5110/6830
Instructor: Alla Borisyuk
Homework 1.2
Due: September 5

1. (EK 1.17. *Annual plants model*) The model for annual plants was condensed in class into a single equation for p_n.

   a) Show that can also be condensed into a single equation for S_n^1.

   b) Explain the term $\alpha\sigma\gamma p_n$ in the equation for p_n.

   c) Let $\alpha = \beta = .001$ and $\sigma = 1$. How big should γ be to ensure that the plants population increases in size?

   d) Show that equation

   $$\gamma > \frac{1}{\alpha\sigma + \beta\sigma^2(1-\alpha)}$$

   gives a more general condition for plant success (Hint: consider $\lambda_1 = \frac{1}{2}(a + \sqrt{a^2 + 4b})$ and show that $\lambda_1 > 1$ implies $a + b < 1$).

2. (EK 1.16) We track the number of red blood cells (RBCs), which should remain roughly constant over time. Let

R_t	=	number of RBCs in circulation on day t
M_t	=	number of RBCs produced by bone marrow on day t
f	=	fraction of RBCs removed each day
γ	=	a production constant

   and suppose they follow the equations

   $$\begin{aligned} R_{t+1} &= (1-f)R_t + M_t, \\ M_{t+1} &= \gamma f R_t. \end{aligned} \tag{1}$$

   a) Try to make sense of these equations (describe in words what is going on) with the particular attention to meaning of the parameter γ and the distinction between R_t and M_t. Are blood cells ready to get to work right after being produced?

   b) Find the eigenvalues and determine their signs and compare their magnitudes.

   c) Show that the TOTAL number of RBCs will remain constant if the larger eigenvalue is equal to 1. What is the value of γ?

   d) In the case from c), what is the other eigenvalue? Describe the resulting dynamics.

3. Consider the Peter Pan model Suppose a population can experience two types of years. In the first $m = 0.5, p = 0.5$ and $\sigma = 0.9$. In the second $m = 2., p = 0.5$ and $\sigma = 0.2$.

a) Find the leading eigenvalue for each population. What would happen to each in the long run?

b) Suppose instead that the two types of years alternate. Find the matrix describing what the population would look like after two years.

c) Find the leading eigenvalue of this two year matrix. Will the population grow or shrink?

d) Explain how this is possible.