

**Section 3.8, problem 6.**

$$x_{t+1} = x_t - \frac{x_t^3 - 20}{3x_t^2}$$

Start with initial guess  $x_0 = 3$ .

$$x_1 = 3 - \frac{3^3 - 20}{3 \cdot 3^2} = 2.7407$$

$$x_2 = x_1 - \frac{x_1^3 - 20}{3 \cdot x_1^2} = 2.7147$$

$$x_3 = x_1 - \frac{x_2^3 - 20}{3 \cdot x_2^2} = 2.7144$$

**Section 3.8, problem 28.**

$$M_t = p(M_t)M_t + 1$$

1. To find an equilibrium we need to solve:

$$M^* = p(M^*)M^* + 1,$$

which is equivalent to finding a zero of

$$f(x) = x - p(x)x - 1.$$

$f(0) = -1 < 0$ , and  $f(10) = 5.69 > 0$ , so by the intermediate value theorem there is an equilibrium of the above system between  $M = 0$  and  $M = 10$ .

2. Start with  $M_0 = 10$ . Then  $M_1 = 4.3109$ ,  $M_2 = 3.5211$ ,  $M_3 = 3.2284$ ,  $M_4 = 3.1039$ ,  $M_5 = 3.0481$ ,  $M_6 = 3.0225$ ,  $M_7 = 3.0107$ ,  $M_8 = 3.0052$ ,  $M_9 = 3.0026$ ,  $M_{10} = 3.0014$ . According to this  $M^*$  is close to 3.

3. NOW do the same thing with the Newton's method.

$$x_{t+1} = x_t - \frac{x_t - p(x_t)x_t - 1}{1 - p'(x_t)x_t - p(x_t)} = x_t - \frac{x_t - 0.9e^{-0.1x_t}x_t - 1}{1 + 0.09e^{-0.1x_t}x_t - 0.9e^{-0.1x_t}}.$$

Start with  $x_0 = 10$ .

Then  $x_1 = 4.3109$ ,  $x_2 = 3.1273$ ,  $x_3 = 3.0020$ ,  $x_4 = 3.0020$ ,  $x_5 = 3.0004$ . This system converges to  $M^* = 3$  much faster.

**p.128, 2**

a.  $y = (3/16)t + 2$ , where  $t$  is time in hours from 5PM on Tuesday and  $y$  is the number of bacteria in hundreds of millions.

b.  $11 = (3/16)t + 2$ . Solving for  $t$ :  $t = 48$ , i.e at 5pm on Thursday.

c. Their culture is described with  $y_2 = (1.4/16)t + 2$ . Need to solve:

$$(3/16)t + 2 = 2((1.4/16)t + 2).$$

$t = 32/.2 = 160$  hours, i.e. at 9am on tuesday a week later.

p.128, 4

a.  $3x_t$ , where time step is 2 hours

b.  $162 \times 10^7$

c.  $x_0 3^t$

d.  $2 \times 10^7 \times 3^t = 10^9$ , i.e  $3^t = 50$ ,  $t = 3.57$  time units or  $t = 3.57 \times 2 = 7.14$  hours

**p.233, 28**

b.  $V(2) = 2$ ,  $V(2.5) = 3.25$ . Secant line  $y = 2 + 2.5(t - 2)$ .

c.  $V'(t) = -2 + 2t$ .  $V'(2) = 2$  thousands of cubic microns per minute.

d. It is growing ( $V'(2) > 0$ )

**p. 234,34**

a.  $P(1) = 1/3$ ;  $P(2) = .4$ . **Average rate of change  $(.4 - 1/3)/1 = 1/15$**

b.  $y = .4 + 0.07(t - 2)$