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Math 1170
Midterm 1 Practice problems

First, read pages 73-76 in the book, these are some examples that we did not have time to consider.

Second, **please, keep in mind that the test may include any of the material that we encountered in class/homeworks, even if it does not directly appear in the practice problems.**

1. The size of a bacterial population b_t is described by the following dynamical system:

$$b_{t+1} = 0.5b_t$$

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a) Find the formula for the solution of

$$b_{t+1} = 0.5b_t$$

with $b_0 = 2$.

b) What do you think will happen to the population if we observe it for a very long time?

c) Find the equilibria of this discrete-time dynamical system algebraically.

2. $f(x) = 3x - 1$, $g(x) = \frac{2}{2-x}$.

a) Find the composition $(g \circ f)(x)$

b) Find $(f \cdot g)(2)$

c) Find the inverse of $f(x)$

d) Plot the graph of $f(x)$

3. We recorded the following data describing the growth of a plant:

Age, a (days)	Mass, M (g)
0.5	2.5
1.0	4.0
1.5	5.5
2.0	7.0
2.5	8.5

a) Plot the mass as a function of age.

b) Notice that this data falls on a line. Find the equation of the line in either point-slope or slope-intercept form

c) Based on the equation of the line, what is your prediction for the mass of a 4-days-old plant?

d) Suppose we also have a different plant type, whose mass is given by

$$N(a) = 2a + 2$$

. On what day will the two plants have equal mass?

4. My height is 5 feet 5 inches (I think). What is it in centimeters (cm)? (1 foot = 12 inches, 1 inch = 2.54 cm.)

5. A discrete-time dynamical system is given by

$$x_{t+1} = x_t^2 + 1.$$

a) What is the updating function?

b) Compute the solution starting from $x_0 = 0$ up to time $t = 3$, by applying the updating function several times.

c) Graph the solution that you found as a function of time.

6. $x_{t+1} = 0.5x_t + 1$.

a) Find the equlibria and study their stability with cobwebbing.

b) Using cobwebbing determine what will happen to the solution with x_0=3 if we wait for a long time? Sketch this soultion as a function of time.

7. Consider

$$w_{t+1} = \frac{aw_t}{1 + w_t}.$$

Find the equilibria analytically. Identify values of the parameter for which there is no equilibrium, for which equilibrium is negative, and for which there is more than one equilibrium.