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Math 1170
Midterm 3 Practice problems

Please, keep in mind that the test may include any of the material that we encountered in class/homeworks, even if it does not directly appear in the practice problems.

The midterm will cover material that we discussed in class from sections 3.3-3.8

1. Consider $f(x) = 2x^3 - 9x^2 + 12x$.

a) Find the local maxima and local minima. Determine which one is which, using the second derivative.

b) Find global maximum and global minimum of $f(x)$ for $0 \leq x \leq 1$

c) Sketch a graph of $f(x)$ based on the above findings

2. Show that the following discrete-time dynamical system has a fixed point b^* such that $0 < b^* < 1$:

$$b_{n+1} = (b_n + 1)e^{-b_n}$$

3. On Sunday we measured a plant and found that it was 2 cm tall. 24 hours later the plant's height was 26 cm.

a) What was the plant's average growth rate? (include units for the rate)

b) What does mean value theorem tell us about the instantaneous growth rate of the plant?

c) What does intermediate value theorem guarantee about the height of the plant?

4. You have a rope of length $2L$. You need to use the rope to build a rectangular enclosure with the largest possible area. What will be the dimensions of your enclosure?

5. Find the following limits:

a) $\lim_{x \rightarrow 3} \frac{e^{-x}}{x-1}$

b) $\lim_{x \rightarrow \infty} \frac{e^{-x}}{e^{-x}-1}$

c) $\lim_{x \rightarrow \infty} \frac{e^{-2x} + 4e^{-x}}{-2e^{-5x} - 3e^{-x}}$

d) $\lim_{x \rightarrow \infty} \frac{\ln(x+5) + x^2 - x^4}{1-x^3}$

e) $\lim_{x \rightarrow 0} \frac{x}{\ln(x+1)}$

6. Find the leading behavior at zero and infinity. Sketch a graph, using the method of matching leading behavior

a) $\frac{3x^2}{1+x^2}$

b) $\frac{1}{1+e^x}$

7. $g(1) = 2$, $g(2) = 3.1$, $g(3) = 2.5$. Estimate $g(1.2)$ and $g(2.8)$, using linear interpolation of $g(t)$.

8. Approximate $f(a) = e^a$ near $a = 1$ by a quadratic polynomial. Use this approximation to estimate $e^{1.2}$