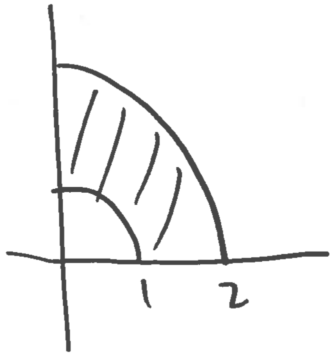
Math 1260 - Quiz 7

1. Evaluate by using polar coordinates

$$\iint_S y \, dA$$

where S is the first quadrant polar rectangle inside $x^2 + y^2 = 4$ and outside $x^2 + y^2 = 1$.



$$\int_0^{\pi/2} \int_1^2 (r \sin\theta) \, r \, dr \, d\theta$$

$$= \int_0^{\pi/2} \frac{r^3}{3}\Big|_1^2 \sin\theta \, d\theta = \frac{7}{3}.$$

2. Using your answer from 1) and a bit of symmetry, find the center of mass for S. (assuming constant density of 1).

$\bar{x} = \bar{y}$ by symmetry

$$\bar{y} = \frac{\iint_S y \, dA}{\iint_S 1 \, dA} = \frac{\frac{7}{3}}{\text{Area}(S)} = \frac{\frac{7}{3}}{\frac{1}{4}(4\pi - \pi)} = \frac{\frac{7}{3}}{\frac{3\pi}{4}} = \boxed{\frac{28}{9\pi}}$$

3. If $z = f(x, y) = \sqrt{R^2 - x^2 - y^2}$ is the z-coordinate for a hemisphere of radius R, then draw a picture of a region S such that $\iint_S f(x, y) \, dA$ computes $1/8$ of the volume of a sphere of radius R. (Hint: Don't integrate anything, this is about how to set something up)