Math 4200 Summer 2008
Homework by Section

Homework 1.1: 1(b), 2(c), 4(a), 6(a), 7, 10* , 11**, 14, 17(a).

*Problem 10: Remark on the nature of the transformation ϕ_z; what kind of linear transformation is it?

Problem 11: Multiplication properties only; note that you need to check Existence **and Uniqueness for identity and inverses.

Homework 1.2: 1(a), 2(b), 4, 5, 8, 10, 11, 12, 14, 18, 19.

Homework 1.3: 1(a), 4(b), 5(a), 6(a), 7(a), 9, 15, 16(abc), 18, 21, 22, 23*, 30(bd), 34.

*Problem 23: you are to prove the statement about how $z \mapsto 1/z$ affects the unit circle.

Homework 1.4: (Recall your Real Analysis I and II)
1, 2(b), 3*, 4*, 5*, 6, 7, 8, 11 (you must prove your assertion!), 13, 16, 18.

*Problems 3, 4, and 5: You can do these in any order; whichever ones you do first, you can use to prove the later ones.

Homework 1.5: 1(ad), 3(b), 7*, 8, 9, 10, 11, 13, 14(abcd), 16, 18(abc), 19, 25, 26, 27, 28**, 29, 31.

*Problem 7 is poorly stated. Better would be (in the language of the differential): Prove that

$$d\left(f^{-1}\right)_{f(a)} = (df_a)^{-1}$$

by considering the composition $f^{-1} \circ f = \mathrm{id}$.

Or, in the language of derivatives (Jacobians): Prove that

$$\left(f^{-1}\right)'(f(z)) = \frac{1}{f'(z)}.$$

Problem 28: The graph of the function $u(x, y) = x^3 - 3xy^2$ is called the **monkey saddle (because it has an extra space for a monkey to put his tail if he sat there). Similar to the (human) saddle $s(x, y) = xy$, the function $u(x, y)$ has a saddle point at $(0, 0)$. However, unlike $s(x, y)$ this saddle point

for $u(x, y)$ is **degenerate**, meaning the matrix of second partials at $(0, 0)$ is singular (this matrix is called the **Hessian** of u at $(0, 0)$).

Homework 1.6: 1(c), 2(abc), 3(a) (there is any easy way), 4, 5, 6, 7, 8*, 10, 14.

*Problem 8: consider the function $f(x, y) = u(x, y) + iv(x, y)$, then what is the function $g(x, y) = u_1(x, y) + iv_1(x, y)$?

Homework 2.1: 2(ac), 3, 5, 10, 11, 12, 13, 14, 15, 16.

Homework 2.2: 1(ad), 2, 3*, 4, 6, 8, 11

*Problem 3: Prove this using the Fundamental Theorem of Calculus

Homework 2.3: 1, 2, 3, 4, 5, 7, 8, 10.

Homework 2.4: 1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 20.

Homework 2.5: 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18.

Homework 3.1: 1, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14.

Homework 3.2: 1(abc), 3, 4, 6, 7(a), 8(b), 11, 13, 15, 18, 20, 22, 23, 24, 25, 26.

Homework 3.3: 1(abd), 4, 5, 7, 8, 10, 11, 13, 17, 18, 20.