

Math 3210-1 Worksheet 2
Not to be turned in.

Sets and Functions

Recall the “Image/Pre-Image Theorem” from class:

Theorem (Image/Pre-Image). Suppose that $f : A \rightarrow B$. Let C, C_1 , and C_2 be subsets of A , and let D, D_1 , and D_2 be subsets of B . Then the following are true:

- (a) $C \subseteq f^{-1}[f(C)]$
- (b) $f^{-1}[f(D)] \subseteq D$
- (c) $f(C_1 \cap C_2) \subseteq f(C_1) \cap f(C_2)$
- (d) $f(C_1 \cup C_2) = f(C_1) \cup f(C_2)$
- (e) $f^{-1}(D_1 \cap D_2) = f^{-1}(D_1) \cap f^{-1}(D_2)$
- (f) $f^{-1}(D_1 \cup D_2) = f^{-1}(D_1) \cup f^{-1}(D_2)$
- (g) $f^{-1}(B \setminus D) = A \setminus f^{-1}(D)$

1. Complete the proofs of the following parts of the Image/Pre-Image Theorem.

Proof. (c) Let $y \in f(C_1 \cap C_2)$. Then there exists a point x in $C_1 \cap C_2$ such that _____ . Since $x \in C_1 \cap C_2$, $x \in$ _____ and $x \in$ _____ . But then $f(x) \in$ _____ and $f(x) \in$ _____, so $y = f(x) \in$ _____ .

(f) Let $x \in f^{-1}(D_1 \cup D_2)$. Then $f(x) \in$ _____, so $f(x) \in D_1$ or $f(x) \in D_2$. If $f(x) \in D_1$, then $x \in$ _____ . If $f(x) \in D_2$ then _____ . In either case, $x \in f^{-1}(D_1) \cup f^{-1}(D_2)$.

Conversely, suppose $x \in$ _____ . Then $x \in f^{-1}(D_1)$ or $x \in f^{-1}(D_2)$. If $x \in f^{-1}(D_1)$, then $f(x) \in$ _____ . If $x \in f^{-1}(D_2)$, then $f(x) \in$ _____ . In either case, $f(x) \in D_1 \cup D_2$, so that $x \in$ _____ .

□

2. Draw pictures to help you understand each part of the Image/Pre-Image Theorem.