Math 3210-1 HW 1

Due Tuesday, June 15, 2004

Logic

1. Write the negation of each statement.
   (a) G is an abelian group.
   (b) The set of prime numbers is finite.
   (c) Jack and Jill are on the hill.
   (d) Three is odd, or four is prime.
   (e) If today is not stormy, then I am riding my bike.
   (f) If f is bounded and linear, then f is continuous.

2. Construct a truth table for each statement.
   (a) $p \implies \sim q$
   (b) $\sim p \ \vee \ q$
   (c) $p \wedge \ \sim p$
   (d) $[\sim q \ \wedge \ (p \implies q)] \implies \sim p$
   (e) $[p \ \wedge \ (p \implies q)] \implies q$
   (f) $[p \implies (q \vee \ \sim q)] \iff \sim p$

3. Indicate whether each statement is true or false.
   (a) 5 is prime and 3 is even.
   (b) 5 is prime or 3 is even.
   (c) $1 > 3$ or 6 is prime.
   (d) If 2 is prime, then 2+2=3.
   (e) If 2+2=3, then 3 is prime.
   (f) If π is rational, then 6 is prime.
   (g) If $2 > 5$ and 3 is prime, then $3^2 = 9$.
   (h) If $12 < 5$ only if 3 is prime, then 4 is odd.

Quantifiers

4. Write the negation of each statement.
   (a) Some apples are blue.
   (b) All dogs have four legs.
   (c) $\exists x > 1 \ni f(x) = 3$.
   (d) $\forall x \in A, \exists y \in B \ni x < y < 1$.
   (e) $\forall x \exists y \ni \forall z, x + y + z \leq xyz$.

5. Determine the truth value of each statement, assuming that x, y, and z are real numbers.
   (a) $\forall x$ and $\forall y, \exists z \ni x + y = z$.
   (b) $\forall x \exists y \ni \forall z, x + y = z$.

(c) $\exists x \ni \forall y, \exists z \ni xz = y$.

(d) $\forall x \exists y \ni \forall z, z > y \implies z > x + y$.

(e) $\forall x$ and $\forall y, \exists z \ni z > y \implies z > x + y$.

The following two problems define certain properties of functions. You are to do two things:

(a) rewrite the defining condition in logical symbolism using $\forall, \exists, \ni$, and $\implies$ or $\iff$, as appropriate; and

(b) write the negation of part (a) using the same symbolism.

6. A function f is *periodic* iff there exists a $k > 0$ such that, for every x, $f(x + k) = f(x)$.

7. A function f is *strictly decreasing* iff for every x and for every y, if $x < y$, then $f(x) > f(y)$.