

Math 2280-1 Review for Final Exam

Below is a list of problem. These are the types of problems you might see on the exam. Certainly, not all of these problems will appear on the exam (there isn't enough time for that); and it is entirely possible that a problem will appear on the exam which does not fit into any of these types.

1. Verify that a given function is a solution to a given DE.
2. Solve $\frac{dy}{dx} = f(x)$ by integration.
3. Solve $\frac{d^2y}{dx^2} = f(x)$ by integration.
4. Solve an IVP.
5. Decide whether or not the IVP

$$\frac{dy}{dx} = f(x, y); \quad y(a) = b$$

has a unique solution.

6. Given a slope field, plot several solution curves.
7. Draw a slope field for a given DE.
8. Draw a set of solution curves for a DE by inspection of the equation.
9. Solve a separable DE.
10. Recognize singular solutions to a DE.
11. Solve a linear 1st order DE using integrating factors.
12. Use substitution methods to transform a DE from any of the following types into a more soluble form:
 - $y' = F(ax + by + c)$, or $y' = F(g(x, y))$ for arbitrary $g(x, y)$.
 - Homogeneous 1st order equations. Note this definition of homogeneous is different than that for linear DEs.
 - Bernoulli equations. Be sure you can recognize a Bernoulli equation.
 - Exact equations. Be sure you know how to tell if an equation is exact. I'm likely not going to tell you.
 - 2nd order DEs with one of the variables missing.
13. Find equilibrium solutions for a given DE.
14. Interpret the physical meaning of fixed points for DEs that arise as physical or biological models.
15. Analyze the stability of fixed points in a given DE.
16. Draw phase diagrams.
17. Find the steady-state solution to a given DE. That is the solution $x(t)$ as $t \rightarrow \infty$.
18. Interpret the physical meaning of steady-state solutions for DEs that arise as physical or biological models.
19. Analyze the dependence on parameters for a given DE.
20. Draw a bifurcation diagram for a given DE.

21. Employ any of the following physical or biological models:
 - Toricelli's Law.
 - Newton's Law of Cooling.
 - Exponential Growth/Decay.
 - Logistic Growth.
 - Doomsday.
 - Generalized Population Growth.
 - Drag forces proportional to a power p of velocity.
 - Variable Gravitational Acceleration, Newton's Law of Gravity.
 - Spring, Mass, Dashpot system.
22. Employ the Principle of Superposition for homogenous linear equations.
23. Solve a 2nd order IVP.
24. Determine whether or not two or more functions are linearly independent.
25. Use the Wronskian of two or more solutions to a DE to determine if they form a basis for the solution space.
26. Find the general solution to a homogeneous linear equation.
27. Using a particular solution y_p to a nonhomogeneous linear n th order DE, and n linearly independent solutions, y_1, \dots, y_n , to its associated homogeneous equation, find the general solution to the nonhomogenous equation.
28. Use the characteristic equation to find the general solution to a linear DE with constant coefficients.
29. Some light proofs.
30. Some written explanations.
31. Definitions. Don't spend too much time studying these, there won't be many.
32. True or False problems. You will need to support your answer.
33. Understand the differences between Overdamped, Underdamped, and Critically Damped mechanical systems.
34. Solve a nonhomogeneous equation by the method of Undetermined Coefficients.
35. Solve a nonhomogeneous equation through Variation of Parameters.
36. Find the natural frequencies of a mechanical system.
37. Understand resonance in mechanical systems.
38. Find the practical resonant frequency in a damped mechanical system.
39. Solve a boundary value problem (endpoint problem) through an eigenvalue method.
40. Compute the deflection curve of a uniform beam under uniform load with various boundary conditions.
41. Compute the deflection curve of a buckled rod with boundary conditions.
42. Determine Existence and Uniqueness for a first-order linear system.
43. Solve a first order linear system.
44. Reduce higher order linear systems to first order linear systems.
45. Use the Wronskian of solutions to determine linear independence.

46. Given n solutions to a homogeneous first order linear system in n variables, find the general solution.
47. Find the solution to a nonhomogenous linear system in n variables.
48. Compute the eigenvalues and associated eigenvectors of a matrix \mathbf{A} . Use these to solve $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$.
49. Compute chains of generalized eigenvectors for defective eigenvalues in order to solve $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$.
50. Compute the fundamental matrix of a linear system.
51. Compute $\exp(\mathbf{A})$ for a square matrix \mathbf{A} .
52. Prove some simple results about square matrices.
53. Compute the matrix exponential solution to $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$.
54. Determine the nature (including type and stability) of the equilibrium solution at the origin for a linear first order system of two variables.
55. Sketch the phase plane near the origin.
56. Compute the Jacobian of an almost linear system at each of its critical points.
57. Determine the nature (including type and stability) of the equilibrium solution, (x_*, y_*) , for an almost linear first order autonomous system of two variables.
58. Sketch the phase plane near each of the critical points.
59. Make inferences regarding an ecological model based on the nature of its critical points, and the local behavior near those critical points.
60. Compute the Laplace Transform of a function $f(t)$ from the definition alone.
61. Compute the Inverse Laplace Transform of a function $F(s)$ from a table of transforms together with theorems regarding the Laplace Transform.
62. Decompose a rational function $F(s)$ into its partial fractions.
63. Convolute two functions f and g by the definition.
64. Solve an IVP through the use of Laplace Transforms.
65. Given a mechanical system, compute its transfer function $W(s)$ as well as its weight function $w(t)$. Use these to find the appropriate responses, $x(t)$, of the system to various forcing functions, $f(t)$, including the unit impulse and unit step responses.
66. Demonstrate basic understanding of linear functionals on a vector space V .