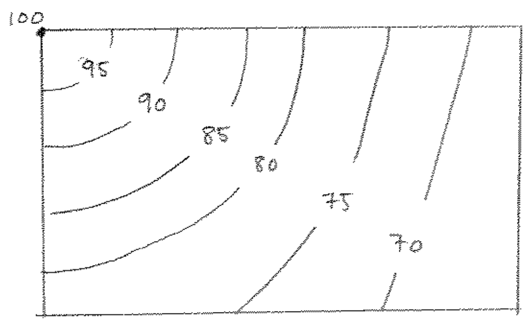
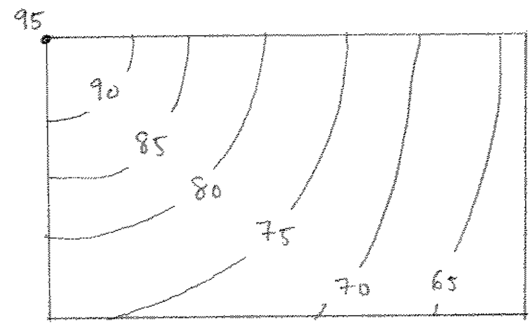
Functions of More Than Two Variables

24. possibly something like:



Surface temps

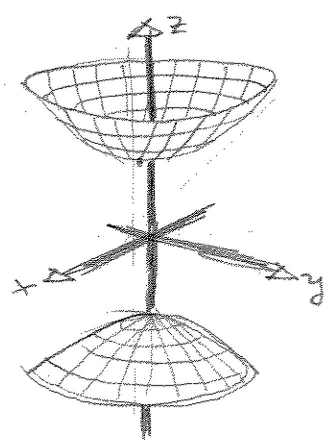


Temps 3 ft. under

25. (a) I (b) II

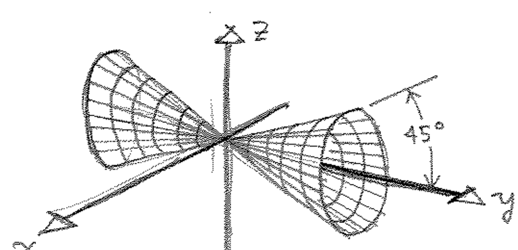
26. the room is heating with the source of the heat in the center of the south wall. There is possibly a heater here which turned on recently.

27. (a) $-x^2 - y^2 + z^2 = 1$: Hyperboloid of two sheets centered on the z-axis.

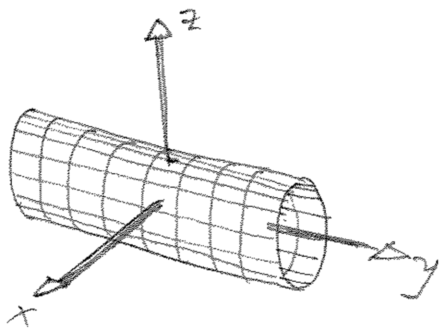


(b) $x^2 - y^2 - z^2 = 1$: Hyperboloid of two sheets on the x-axis. (Turn the above picture sideways).

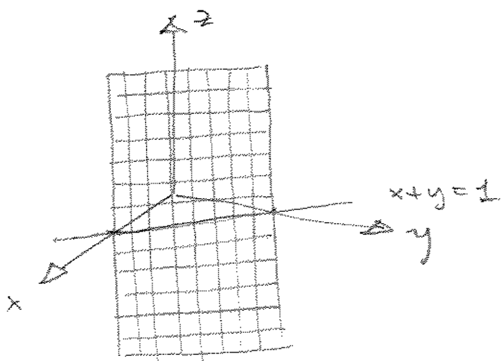
(c) $-x^2 + y^2 - z^2 = 0$: circular cone along y-axis with an angle of 45° from the axis.



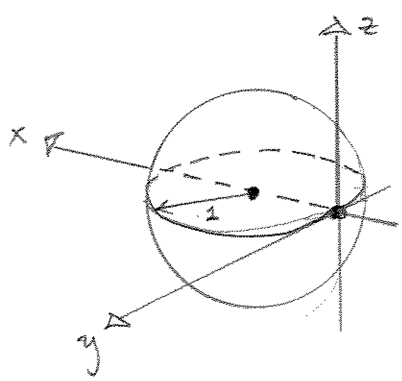
27. (d) $x^2+z^2=1$: Circular cylinder, axis along y-axis, radius 1.



(e) $x+y=1$: Vertical plane ($\perp$ to xy-plane) intersecting xy-plane along the line $y=1-x$.



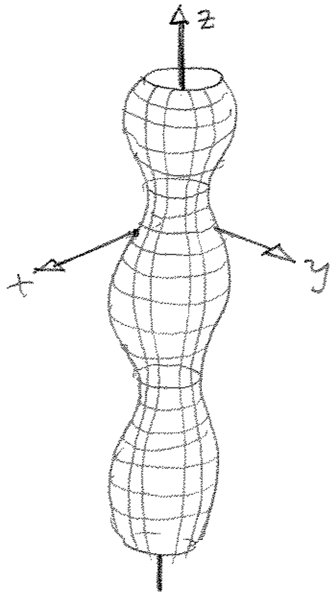
(f) $(x-1)^2+y^2+z^2=1$: Sphere of radius 1, centered at (1,0,0)



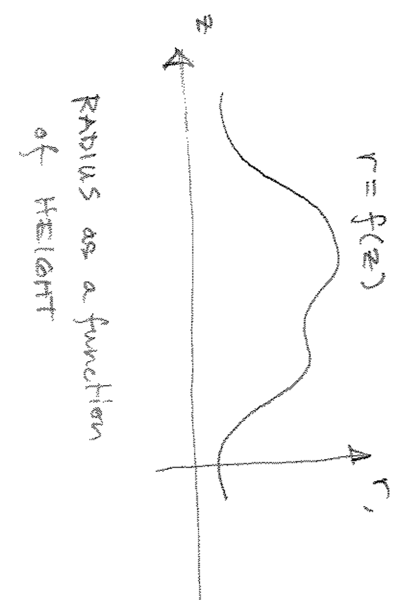
Note: I drew this with an unusual vantage point so you can see that the sphere is tangent to the y- and z-axes.

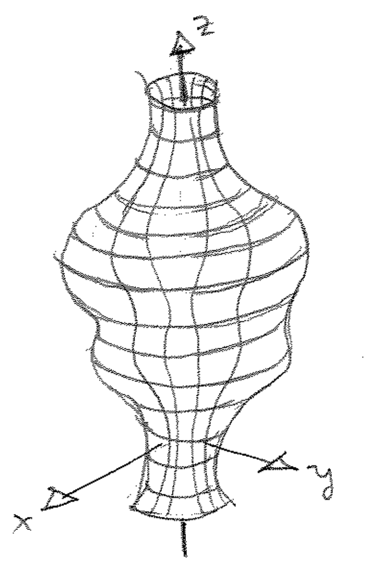
28. $x^2 + y^2 = (2 + \sin z)^2$.

This is a surface of revolution about the z-axis. The radius of this surface is $2 + \sin z$, thus periodic in z. It could be described as a "wavy cylinder".



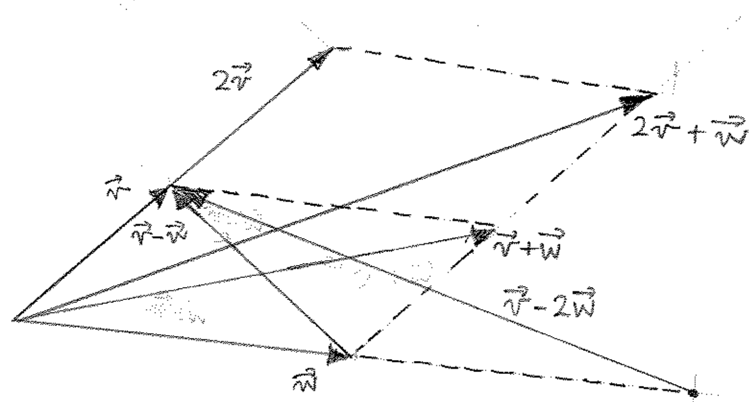
$x^2 + y^2 = [f(z)]^2$. Similar to above, but the cylinder's radius at height z is given by $f(z)$.





DISPLACEMENT VECTORS

29.



30. (a) $\hat{\imath}+4\hat{\jmath}$

(b) $\hat{k}$

(c) $-\hat{\imath}$

(d) $\hat{\imath}+\hat{k}$

31. $\vec{q}=4\vec{v}$ so $\vec{q}$ and $\vec{v}$ are parallel.

Note that $\vec{u}=-2\vec{w}$ so $\vec{u}$ and $\vec{w}$ are "antiparallel".

32. (a) $\|\vec{z}\|=\sqrt{11}$

(b) $\vec{v}+\vec{z}=\hat{\imath}-\hat{\jmath}$

(c) $2\vec{w}+\vec{x}=21\hat{\jmath}$

(d) $\|\vec{y}\|=\sqrt{65}$

(e) $\|\vec{y}-\vec{x}\|=\|6\hat{\imath}-16\hat{\jmath}\|=\sqrt{292}=2\sqrt{73}$

33. $\frac{2}{\sqrt{6}}\hat{\imath}-\frac{2}{\sqrt{6}}\hat{\jmath}+\frac{4}{\sqrt{6}}\hat{k}$ OR $\frac{\sqrt{6}}{3}\hat{\imath}-\frac{\sqrt{6}}{3}\hat{\jmath}+\frac{2\sqrt{6}}{3}\hat{k}$

Vectors in General

34. (a) Scalar (though <u>displacement</u> is a vector).

(b) Scalar

(c) Vector

(d) Scalar

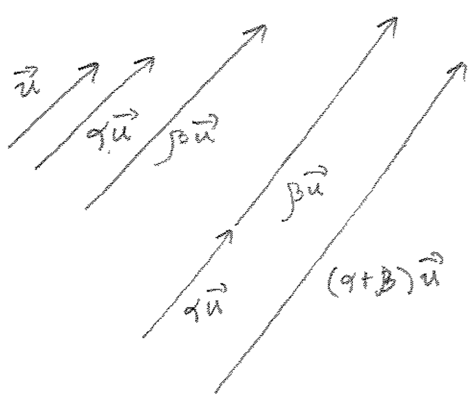
35. See Attached Page after #36.

36. (a) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

First travelling along $\vec{u}$ and then along $\vec{v}$ is the same as first travelling along $\vec{v}$ and then travelling along $\vec{u}$.

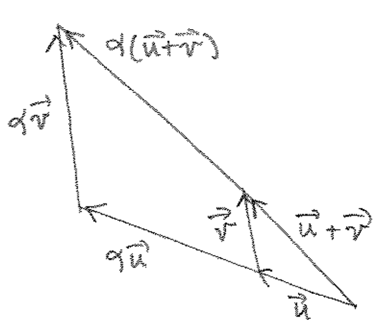
(This is a property of our space being "flat").

(b) $(\alpha + \beta)\vec{u} = \alpha\vec{u} + \beta\vec{u}$



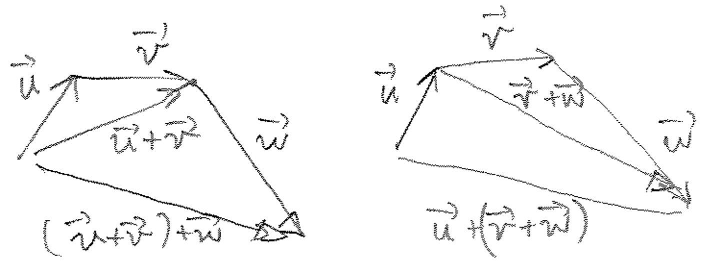
If we travel in the direction of $\vec{u}$, but α times as far, then travel β times as far in the same direction (left hand side), that is equivalent to travelling in that direction a distance of $(\alpha + \beta)$ times the length of $\vec{u}$.

(c) $\alpha(\vec{u} + \vec{v}) = \alpha\vec{u} + \alpha\vec{v}$



Traveling in the $\vec{u}$ direction α times as far, followed by travel in the $\vec{v}$ direction α times as far (left hand side) is equivalent to heading directly in the $\vec{u} + \vec{v}$ direction but α times as far as $\|\vec{u} + \vec{v}\|$. This is similarity of triangles!

36 (d) $(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$



Think of $(\vec{u}+\vec{v})$ and $(\vec{v}+\vec{w})$ as two "shortcuts". If we use one of the shortcuts, it doesn't matter which one — we still get to where we are going.

(e) $\alpha(\beta\vec{u}) = (\alpha\beta)\vec{u}$

$\beta\vec{u}$ is a vector in the $\vec{u}$ direction but β times as long (possibly negative). $\alpha(\beta\vec{u})$ then is α times as long as that, and in the same direction. So the result is a vector in the $\vec{u}$ direction which is $(\alpha\beta)$ times as long. This is just a scalar property.

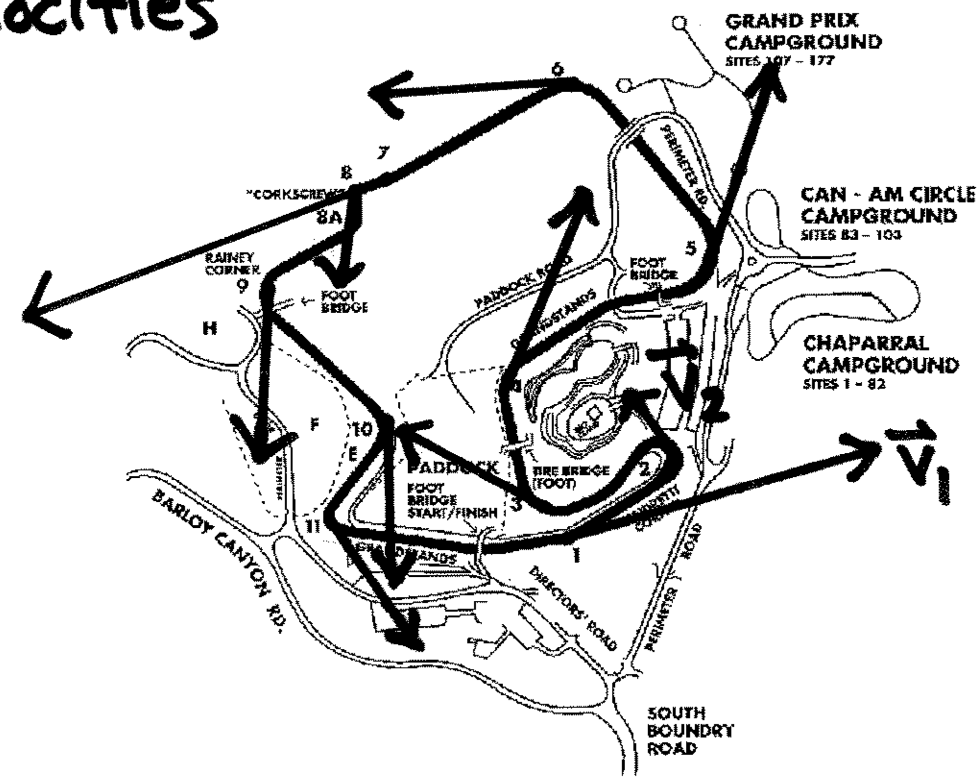
(f). $\vec{v}+\vec{0}=\vec{v}$. Travel along $\vec{v}$ and then go nowhere. This is equivalent to just travelling along $\vec{v}$.

(g) $1\cdot\vec{v}=\vec{v}$ Travel in the $\vec{v}$ direction a distance of 1 times the magnitude of $\vec{v}$. this is the same as travelling along $\vec{v}$.

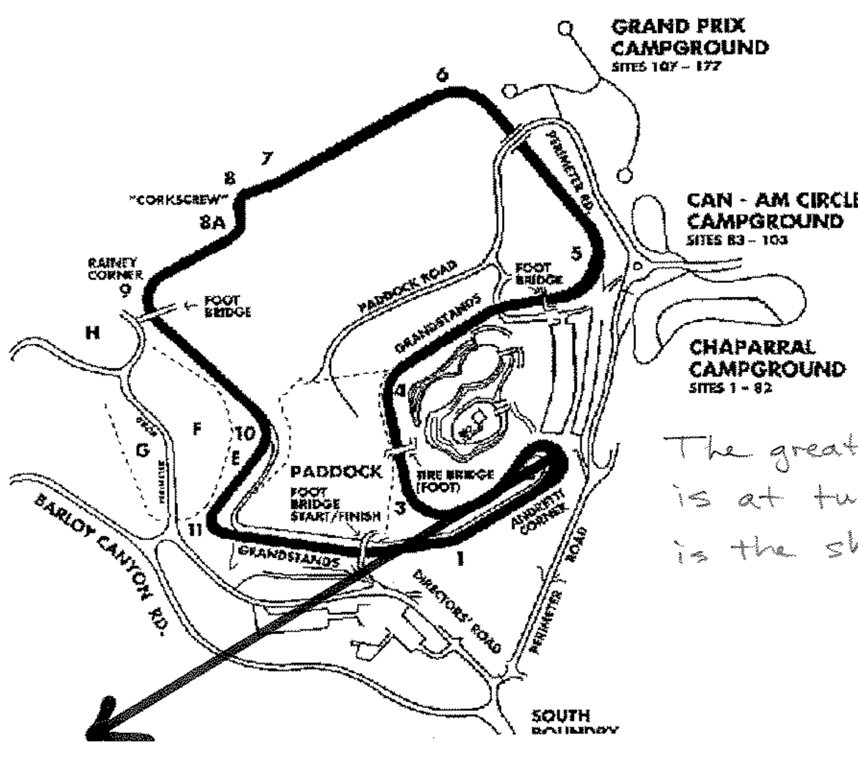
#35

1995 LAGUNA SECA
Track Map

Velocities



1995 LAGUNA SECA
Track Map



The greatest acceleration is at turn 2. This is the sharpest turn.

THE DOT PRODUCT

37. $\vec{a}\cdot\vec{b} = 14$

38. $\vec{u} = \hat{\imath} + \hat{\jmath} + \hat{k}$ $\qquad \|\vec{u}\| = \|\vec{v}\| = \sqrt{3}$

$\vec{v} = \hat{\imath} - \hat{\jmath} - \hat{k}$

$\vec{u}\cdot\vec{v} = -1$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = -\frac{1}{3} \implies \theta \approx 1.91 \text{ radians}$$

$$\approx 109.5^\circ$$

39. $\pi(x-1) = (1-\pi)(y-z) + \pi$

$\pi x - \pi = (1-\pi)y - (1-\pi)z + \pi$

$\pi x + (\pi - 1)y + (1-\pi)z = 2\pi$

$\vec{N} = \pi\hat{\imath} + (\pi-1)\hat{\jmath} + (1-\pi)\hat{k}$ is a normal vector.

40. We take the dot product:

$$[(\vec{b}\cdot\vec{c})\vec{a} - (\vec{a}\cdot\vec{c})\vec{b}]\cdot\vec{c} = (\vec{b}\cdot\vec{c})\vec{a}\cdot\vec{c} - (\vec{a}\cdot\vec{c})\vec{b}\cdot\vec{c} = 0$$

Thus $\left[(\vec{b}\cdot\vec{c})\vec{a} - (\vec{a}\cdot\vec{c})\vec{b}\right] \perp \vec{c}$.

THE CROSS PRODUCT

41. $\hat{k} \times \hat{j} = -\hat{\imath}$

42. No. $\vec{u} \times \vec{u}$ is the zero vector, $\vec{u} \cdot \vec{u}$ is a scalar.

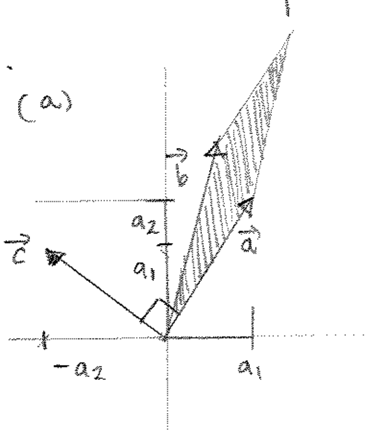
43. (a)
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{\imath} + \hat{j} + \hat{k}$$

(b)
$$\vec{a} \times \vec{b} = \begin{vmatrix} \cdot & \cdot & \cdot \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -2\hat{\imath} + 2\hat{j}$$

(c)
$$\vec{a} \times \vec{b} = -\hat{\imath} \times (\hat{j} + \hat{k}) = -\hat{k} + \hat{j} = \hat{j} - \hat{k}.$$

(d)
$$\vec{a} \times \vec{b} = \begin{vmatrix} \cdot & \cdot & \cdot \\ 2 & -3 & 1 \\ 1 & 2 & -1 \end{vmatrix} = \hat{\imath} + 3\hat{j} + 7\hat{k}$$

44. (a)



(b) $\vec{a} \perp \vec{c}$ and $\|\vec{c}\| = \|\vec{a}\|$.

(c) $\vec{c} \cdot \vec{b} = a_1 b_2 - b_1 a_2$
$= \|\vec{a} \times \vec{b}\|$

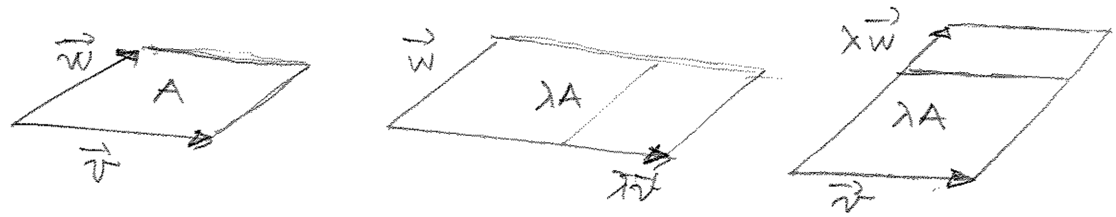
(d) $\vec{c} \cdot \vec{b} = \|\vec{a} \times \vec{b}\|$ and $\|\vec{a} \times \vec{b}\|$ is (by definition) the area of the parallelogram formed by $\vec{a}$ and $\vec{b}$.

Also note that the area of this parallelogram is (base) X (height). If the base is $\|\vec{a}\| = \|\vec{c}\|$ then the height is $\|b\| \cos\theta$ where θ is the angle between $\vec{b}$ and $\vec{c}$. Then $\vec{c} \cdot \vec{b} = \|\vec{c}\| \, \|b\| \cos\theta$.

(e) Clear.

$\lambda > 0$

45. Look at $(\lambda\vec{v}) \times \vec{w}$ and $\vec{v} \times (\lambda\vec{w})$. In each case, the parallelogram is in the same plane as the parallelogram for $\vec{v} \times \vec{w}$. Therefore, the direction of $(\lambda\vec{v}) \times \vec{w}$ or $(\vec{v} \times \lambda\vec{w})$ should be the same as for $\vec{v} \times \vec{w}$ (or opposite). Furthermore, in each case, one of the sides of the parallelogram is scaled by λ, so the area is also scaled by λ.



therefore, in either case, the result is λ times the vector $\vec{v} \times \vec{w}$.

The cases $\lambda < 0$, $\lambda = 0$ are similar.