Some exercises on $\mathbb{H}^2$

1. Let T be an isometry of $\mathbb{H}^2$. Define the *translation length* of T to be
$$\inf_{z \in \mathbb{H}^2} d(z, T(z)).$$
Show that if T is parabolic, then the translation length of T is 0.

2. Prove that for any pair of disjoint geodesics in $\mathbb{H}^2$ that do not meet on $\partial_\infty \mathbb{H}^2$ there exists a unique geodesic that meets each orthogonally.

3. Consider two sets of distinct points $\{z_1, z_2, z_3\}$ and $\{w_1, w_2, w_3\}$ in $\mathbb{R} \cup \{\infty\}$. Construct a linear fractional transformation taking one set to the other.

4. Choose three distinct points in $\partial_\infty \mathbb{H}^2$. The region bounded by the three geodesics determined by these points is called an *ideal triangle*. Compute the hyperbolic area of an ideal triangle. (Hint: compute the area of the ideal triangle with vertices -1, 1, and ∞ and then use number 3.)