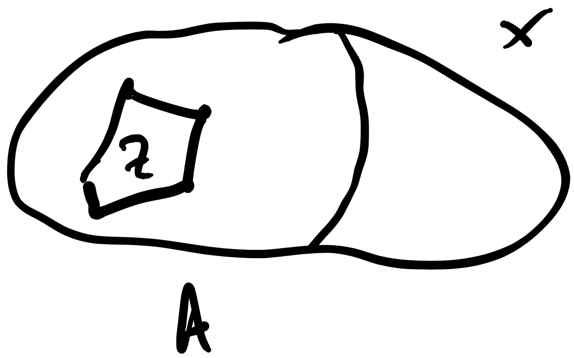
Excision $Z \subset A \subset X$, $\overline{Z} \subset \operatorname{int} A$

$$\Rightarrow H_n(X-Z, A-Z) \xrightarrow{\cong} H_n(X,A) \quad \forall n.$$



$\mathcal{U} = \{A, B = X - Z\}$

$\mathcal{U}$ cover of X, $\bigcup_{U \in \mathcal{U}} \operatorname{int} U = X$

$C^{\mathcal{U}}(X) \subset C(X)$

$\langle \sigma : \Delta^n \to X \mid \operatorname{Im}(\sigma) \subset U$ for some $U \in \mathcal{U} \rangle$.

Thm $H_n^{\mathcal{U}}(X) \xrightarrow{\cong} H_n(X) \quad \forall n.$

Pf of excision $C_n(A+B) := C_n^{\mathcal{U}}(X)$, $\mathcal{U} = \{A, B\}$

$$C_n(X) \supseteq C_n(A+B) \supset C_n(A)$$

LES of this triple:

$$H_n(A+B,A) \to H_n(X,A) \to \underbrace{H_n(X, A+B)}_{\substack{0 \text{ by LES of } (X,A+B) + \text{Thm}}} \to H_{n-1}(A+B, A) \to \cdots$$

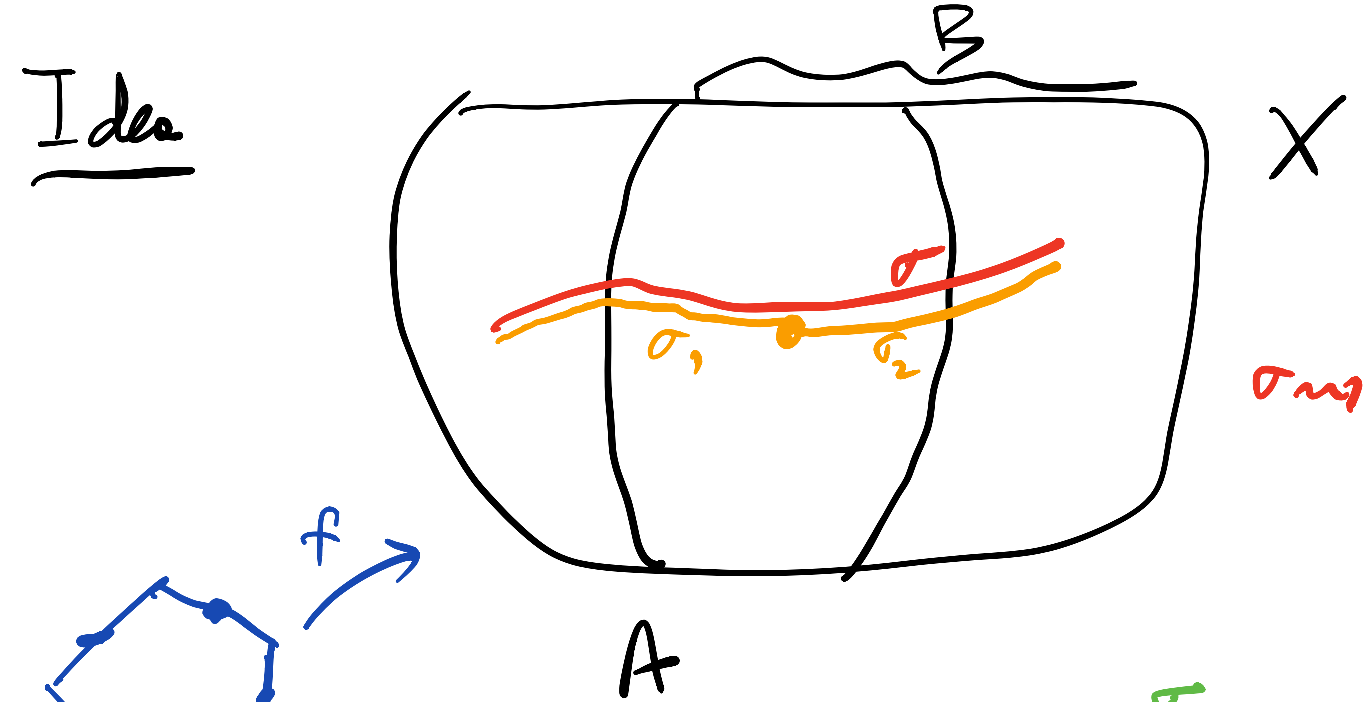
$$\xrightarrow{\cong} \quad \to 0 \to \quad \xrightarrow{\cong} \quad \to$$

$$\Rightarrow H_n(A+B, A) \xrightarrow{\cong} H_n(X, A) \leftarrow \text{excision!}$$

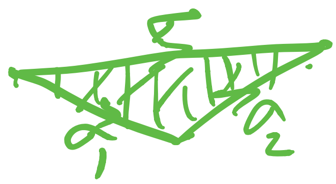
$$H_n(B, A\cap B) = $$

$$C(A+B)/C(A) = C(B)/C(A\cap B)$$

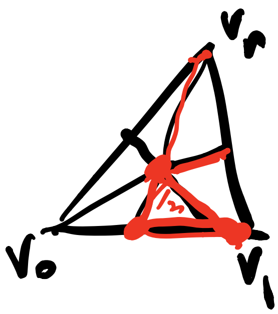
Idea



$\sigma \rightsquigarrow \sigma_1 + \sigma_2$



Barycentric subdivision



$$b = \frac{1}{n+1} \sum v_i.$$

Decomposition of Δ^n into simplices

$[b, w_0, \ldots, w_k]$

where $w_0, \ldots, w_k$ are vertices

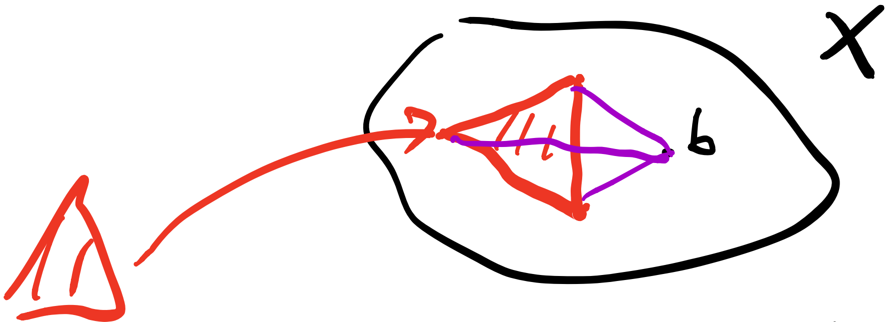
of a simplex in the barycentric subdivision of a face.

Lemma $\operatorname{diam}[b, w_0, \ldots, w_k] \leq \frac{n}{n+1} \cdot \operatorname{diam} \Delta^n$

Warmup Linear chains

X = convex set in $\mathbb{R}^n$

$LC_n(X)$ = complex of linear simplices in X



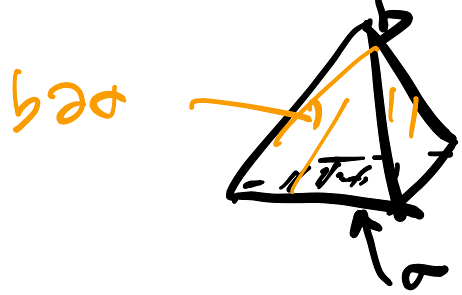
$$\cdots \xrightarrow{\partial} LC_0(X) \xrightarrow{\partial} \underset{\underset{\mathbb{Z} = \langle [\emptyset] \rangle}{\parallel}}{LC_{-1}(X)} \to 0$$

Fix $b \in X$

$\rightsquigarrow \quad b : LC_n(X) \to LC_{n+1}(X)$

$$[v_0, \ldots, v_n] \longmapsto [b, v_0, \ldots, v_n]$$

Claim $\partial b + b \partial = \mathbb{1} \qquad (0 \simeq \mathbb{1})$



$= b \cdot \sigma$

$\partial b\sigma = \sigma \cup b\partial\sigma$

$$\partial[b, v_0, \dots, v_n] = [v_0, \dots, v_n] - b\partial[v_0, \dots, v_n]$$

Subdivision of linear chains

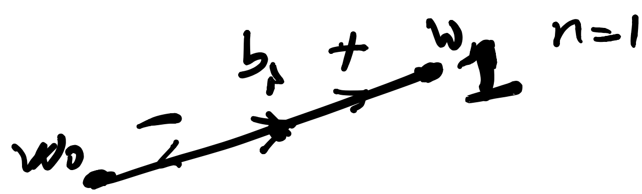
$$S : LC_n(X) \to LC_n(X)$$

$S([\phi]) = [\phi] \qquad n = -1$

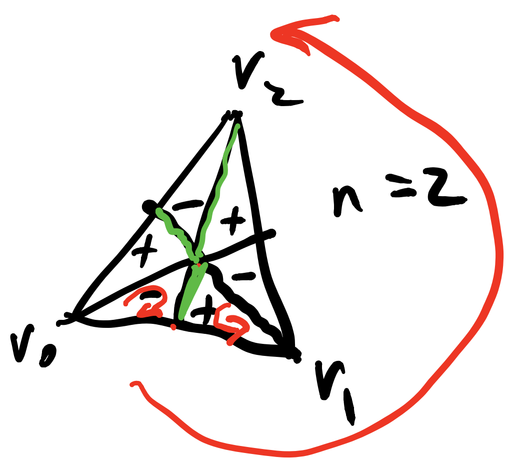
$S([w]) = [w] \qquad n = 0$

$$S(\underbrace{[w_0, \dots, w_n]}_{\lambda}) = b_\lambda(\underbrace{S\partial\lambda})$$

barycenter of λ



$S([w_0, w_1]) = [b_\lambda, w_1] - [b_\lambda, w_0]$



S is a chain morphism

$\partial S = S\partial$

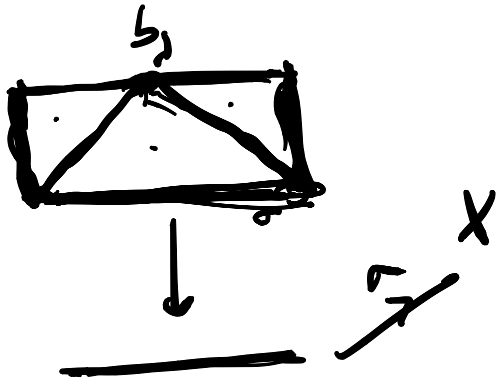
$T: LC_n(X) \to LC_{n+1}(X)$ chain homotopy between S & $\mathbb{1}$.

<u>Cor:</u> S_* is $\mathrm{id}: LH_n(X) \to LH_n(X)$

$T = 0 \quad n = -1$

$T([w]) = [w, w]$

$T\lambda = b_\lambda(\lambda - T\underline{\partial\lambda})$

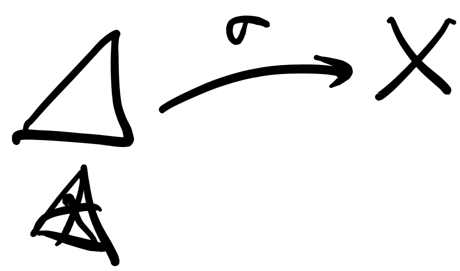


$$\partial T + \underbrace{T\partial}_{\text{vertical part of } \partial} = \underset{\text{bottom}}{\mathbb{1}} - \overset{\text{top}}{S}$$

Subdivision of singular chains

$S: C_n(X) \to C_n(X)$, X arbitrary space.

$S\sigma = \sigma_\# S\Delta^n$



S chain map, chain homotopic to $\mathbb{1}$.

$$T\sigma = \sigma_{\#} T \Delta^n$$

<u>Cor:</u> $S_*: H_n(X) \to H_n(X)$ is identity!

<u>Pf that</u> $H_n^{al}(X) \to H_n(X)$ <u>is onto.</u>

$[z] \in H_n(X)$, z cycle, $z = \sum_{i=1}^{N} n_i \sigma_i$

$[Sz] = [z]$

Lebesgue covering $\Rightarrow \exists k$ s.t. $S^k z \in C_n^{al}(X)$

$[S^k z] = [z]$ in $H_n(X)$

$[S^k z] \in H_n^{al}(X)$ so onto ✓.

1-1 is similar.