Math 5520 Homework 6

Folding

1. Let H be the subgroup of $F_2 = \langle a, b \rangle$ generated by $b, a^2, ab^2a, ababa$.

   (a) Find the immersion $\Gamma_H \hookrightarrow R$ representing H.

   (b) Does $a\bar{b}a\bar{b}a$ belong to H? Does ab^3a?

   (c) Is b^2 conjugate into H? Is a?

   (d) What is the index of H in F_2?

2. Let H be the subgroup of $F_3 = \langle a, b, c \rangle$ generated by a^2, ab, acb.

   (a) Find the immersion $\Gamma_H \hookrightarrow R$ representing H.

   (b) Does aca belong to H? Does b?

   (c) Is b^2 conjugate into H? Is a?

   (d) What is the index of H in F_3?

3. Let
   $$f : F_2 = \langle x, y \rangle \to F_2 = \langle a, b \rangle$$
   be defined by
   $$x \mapsto abbab, \; y \mapsto bababbab$$
   Is f an isomorphism?

4. Let H, K be subgroups of $F_2 = \langle a, b \rangle$ as follows:
   $$H = \langle a, b^2 \rangle$$
   and
   $$K = \langle ba, ab^3a \rangle$$
   Compute $H \cap K$.