

Math 5520 Additional Homework

Center of free groups

Recall that the *center* of a group G is

$$Z(G) = \{z \in G \mid zg = gz \text{ for every } g \in G\}$$

The center is always a subgroup of G ; in fact it is a normal subgroup (you may want to prove it as an algebra exercise).

Exercise. Show that $Z(F_2)$ is the trivial subgroup.

The same is true for all free groups of rank ≥ 2 , but for simplicity we work with rank 2.

Hint: Assume $1 \neq z \in Z(F_2)$. Write z as a reduced word in the basis elements. Depending on this reduced word find an explicit $g \in F_2$ that does not commute with z .