

Aaron Bertram/Curriculum Vita

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Education:

PhD Mathematics June, 1989 UCLA (Robert Lazarsfeld, Advisor)
MA Mathematics December, 1985 UCLA
BA Mathematics June, 1984 Harvard University

Positions Held:

7/00- present	Professor	University of Utah
7/12-6/13	Director	Math for America-Utah
7/05-6/11	Chair/Mathematics Department	University of Utah
8/00-11/00	Visiting Professor	University of Nice
7/95-6/00	Associate Professor	University of Utah
7/92-6/95	Assistant Professor	University of Utah
7/89-6/93	Benjamin Peirce Asst Professor	Harvard University
7/84-6/89	Teaching and Research Assistant	UCLA

Fellowships and Awards:

7/13-6/18 NSF Geometry RTG Grant (Co PI with Bromberg and deFernex)
7/09-6/12 NSF Research Grant
7/05-6/08 NSF Research Grant
7/06-6/11 NSF Departmental VIGRE Grant (Co PI with Dobson)
7/02-6/05 NSF Research Grant (Co PI with Herb Clemens)
7/99-6/02 NSF Research Grant (Co PI with Herb Clemens)
7/95-6/98 NSF Research Grant
9/94-6/96 Sloan Research Fellowship
7/92-6/95 NSF Research Grant
9/89-6/92 NSF Postdoctoral Research Fellowship
9/88-6/89 Sloan Doctoral Dissertation Research Fellowship

External Funding

Recent (Post-2000) Funding Details.

NSF Research Grants. These are the standard “personal” grants for research mathematicians. They include travel support and summer salary for the PI, as well as some undergraduate and graduate student and postdoctoral scholar support, as well as expenses for visitors with the goal of furthering the PI’s specified research project.

1999-2002. DMS 9970412. Co-PI with Herb Clemens. Award: \$212,521.
Families of Curves on Higher Dimensional Complex Projective Manifolds.

2002-2005. DMS 0200895. Co-PI with Herb Clemens. Award. \$218,459.
Questions Related to Curves on Complex Projective Manifolds.

2005-2009. DMS 0501000. Sole PI. Award. \$569,250.
Algebraic Geometry Inspired by Physics.

2009-2014. DMS 0901128. Sole PI. Award. \$286,552.
Bridgeland Moduli of Derived Objects on Algebraic Surfaces.

NSF Training Grants. These are much larger grants meant to benefit a large research group or, in the first case, the entire Mathematics Department. They include very significant undergraduate, graduate and postdoctoral support as well as support for summer mini-courses, workshops and seminars with the goal of transforming research and teaching within the department.

2006-2012. DMS 0602219. Co-PI with Dobson. Award. \$4,483,045.
Vertical Integration in Mathematics at the University of Utah

2013-2018. DMS 1246989. Co-PI with Bromberg/deFernex. Award. \$2,431,178.
Algebraic Geometry and Topology at the University of Utah

NSF Conference Grants. These grants defray the costs of institutes or conferences organized by the PIs.

2005. DMS 0456683. Co-PI with Four Collaborators. Award. \$149,715.
(Seattle) Summer Institute on Algebraic Geometry

2007. PHY 0652421. Co-PI with Four Collaborators. Award. \$4,000.
A Summer Mini-Course on Mathematics and String Theory

2010-2015. DMS 0955038. Co-PI with Three Collaborators. Award. \$286,742.
Western Algebraic Geometry Seminar-Five Year Plan

Research

Journal publications are the customary vehicle for dissemination of research in mathematics. These are necessarily technical and abstract in nature and more difficult to explain to a general audience than research in the other sciences. I have ranked my publications in this CV according to the number of citations, presenting only the sixteen most highly cited (according to Google Scholar) as of March, 2014. Of course, this biases the list toward older research.

Most Significant Publications (in decreasing order of citations)

1. *Quantum Schubert Calculus*, *Advances in Math*, **128** 2 (1997), 289-305.

Schubert calculus is a tool in enumerative geometry dating back to the 19th century. In this paper I used the recently developed technology for counting rational curves to update the Schubert calculus via a “quantum deformation.”

2. *Gromov invariants for holomorphic maps to Grassmannians*, *Journal of the American Mathematical Society*, **9** 2 (1996), 529-571.
(joint with G. Daskalopoulos and R. Wentworth).

This was the first paper to compute the curve-counting invariants that later became known as *Gromov-Witten invariants*. In it, we counted curves of genus zero and one on Grassmannians satisfying incidence conditions.

3. *Vanishing theorems, a theorem of Severi, and the equations of projective varieties*, *Journal of the American Mathematical Society* **4** 3 (1991), 587-602.
(joint with L. Ein and R. Lazarsfeld).

We applied the Kawamata-Viehweg vanishing theorem to obtain detailed information about smooth projective varieties that are the common zero locus of equations of relatively low degree in projective space.

4. *Moduli of rank-2 vector bundles, theta divisors, and the geometry of curves in projective space*, *Journal of Differential Geometry* **35** (1992), 429-469.

This was essentially my PhD thesis, in which I completed a map from projective space to the space of stable rank two vector bundles on a curve. This inspired work of Michael Thaddeus (now at Columbia University) and a mini-industry of studying the stability of pairs; a vector bundle with a section.

5. *Another way to enumerate rational curves with torus actions*, *Inventiones Mathematicae*, **142** (2000) 487-512.

I applied localization in a novel way to compute the numbers of rational curves of all degrees in a quintic hypersurface in projective four-space. This method has been more widely used in other settings than the better known methods due to research groups at Harvard and Berkeley, who have the priority on the result.

6. *Quantum multiplication of Schur polynomials*, Journal of Algebra **219** (1999) 727-746. (joint with I. Ciocan-Fontanine and W. Fulton)

The multiplication of Schur polynomials is a beautiful classical combinatorial prescription describing the algebra of symmetric polynomials. In this paper, we figured out a similar combinatorics behind the multiplication that takes place in Quantum Schubert Calculus.

7. *Two proofs of a conjecture of Hori and Vafa*, Duke Math Journal, **126** 1 (2005), 101-136. (joint with I. Ciocan-Fontanine and B. Kim).

To finish off the counts of rational curves on a Grassmannian begun in the Quantum Schubert Calculus paper, a difficult localization computation needed to be made. In this work, we made the computation and produced a closed formula for the result, besting a research team at Harvard that was attempting to solve the same problem.

8. *Hilbert polynomials of moduli spaces of rank 2 vector bundles II*, Topology **32** 3 (1993), 599-609 (joint with A. Szenes).

A conjecture from physics, known as the *Verlinde formula* for the dimension of the spaces of higher-rank theta functions was a hot topic while I was a postdoc. In this paper, Szenes and I polished this off in rank two with an application of the Atiyah-Bott localization theorem and some clever reductions.

9. *Towards a Schubert calculus for maps from a Riemann surface to a Grassmannian*, International Journal of Math., **5** 6 (1994), 811-825.

I took the Quantum Schubert Calculus which counted rational curves on a Grassmannian and showed how to use it to compute some of the Gromov-Witten invariants of curves of higher genus. This was, however, completely overshadowed by the Fields-medal work of Maxim Kontsevich that appeared at roughly the same time.

10. *Surjectivity of Gaussian maps for line bundles of large degree on curves*, Lecture Notes in Math **1479** (1991), 15-25 (joint with L. Ein and R. Lazarsfeld).

The Gaussian map is an interesting higher-order multiplication map for sections of line bundles on a curve. Surjectivity of the Gaussian map was of interest in relation to the question of whether a curve did or did not lie on a K3 surface.

11. *On stable rank two vector bundles with canonical determinant and many sections*, Marcel-Dekker Lecture Notes **200** (1998), 259-270 (with B. Feinberg).

We found a natural setting in higher rank in which the classical Brill-Noether theory for line bundles on a curve seems to generalize in the expected way. There are still some interesting open questions posed in this paper, which appeared at about the same time as the paper of Shigeru Mukai, which listed even more examples in support of the generalization.

12. *Stable pairs and stable parabolic pairs*, Journal of Algebraic Geometry **3** (1994), 703-724.

I developed some of the Thaddeus theory for vector bundles with a section on a curve in the setting of parabolic bundles, in support of a second paper that I was writing (which, amusingly, has fewer citations), in which the work with Szenes on the Verlinde formula was extended to parabolic bundles.

13. *Gromov Witten invariants for abelian and non-abelian quotients*, Journal of Alg Geometry **17** (2008), 275-294 (joint with I. Ciocan-Fontanine and B. Kim).

Abelian quotients (essentially toric varieties) are relatively easy to study. Non-abelian quotients (by a reductive group) are difficult to study, from the point of view of counting rational curves. In this paper, we showed how to reduce the difficult to the easy, with an application to computing the invariants of flag varieties, once again beating out a research team at Harvard that was working on the same problem.

14. *Bridgeland-stable moduli spaces for K -trivial surfaces*, Journal of the European Mathematical Society (JEMS) **15** (2013), **1** 1-38 (joint with D. Arcara).

Via string theory, Douglas and Bridgeland realized that “classical” moduli of vector bundles on algebraic surfaces was inadequate to capture all the parameter spaces that were appearing for D-branes in the physics literature. Bridgeland conjectured that nevertheless, the classical moduli (of the 1960’s and 1970’s) could contain within them the information to generate these parameter spaces via the “minimal model program.” In this paper, with my postdoc, we verified Bridgeland’s prediction in one case, and our techniques have since been seen to be applicable in many more cases.

15. *New recursions for genus-zero Gromov-Witten invariants*. Topology **44** **1** (2005), 1-24 (joint with H. Kley).

It was known that it was theoretically possible to compute the quantum products (analogous to the quantum Schubert calculus) for hypersurfaces and complete intersections of low degree in projective space. Holger (my postdoc) and I were interested in actually computing these products. We used localization to do so, generating dozens of interesting examples. This work paralleled results by Lee and Pandharipande (at Cal Tech) who used it for a different application.

16. *The minimal model program for the Hilbert scheme of points on the projective plane and Bridgeland stability*. Advances in Math **235** (2013), 580-626 (joint with D. Arcara, I. Coskun and J. Huizenga).

This nicely bookends the publications, constructing a sequence of models of the configuration space of points on the plane that I expect to be eventually as highly influential as the Quantum Schubert Calculus.

Teaching

Teaching has always been a source of deep satisfaction for me. Some courses are “favorites” that I return to again and again, for example the Foundations of Algebra for Teachers, for which I have developed a complete set of lecture notes (available on-line). The Introduction to Algebraic Geometry is another favorite, in which I have introduced several generations of graduate students to my mathematics specialty. Teaching teachers has been an opportunity for me to learn about the important differences between teaching in higher education versus secondary school.

High School Students

- High School Summer Program in Mathematics
- High School Math Circles

Undergraduate Teaching

- Math 1030 Introduction to Quantitative Reasoning
- Math 1040 Introduction to Statistics and Probability
- Math 1210 Calculus I
- Math 1220 Calculus II
- Math 1250 AP Calculus I
- Math 1260 AP Calculus II
- Math 1310 Engineering Calculus I
- Math 2210 Calculus III
- Math 2270 Linear Algebra
- Math 4400 Number Theory
- Math 4800 Undergraduate Research Topics
- Math 5310 Introduction to Modern Algebra I
- Math 5320 Introduction to Modern Algebra II
- Math 5405 Cryptography

Teaching of Secondary School Teachers

- Math 3100 Foundations of Geometry
- Math 4030 Foundations of Algebra
- Math 5900 Topics in Algebra
- Math 6080 Topics in Contemporary Math
- Teachers Math Circles and Masters Projects for Teachers

Graduate Teaching

- Survival Skills for Graduate Students
- Math 6130 Introduction to Algebraic Geometry I
- Math 6140 Introduction to Algebraic Geometry II
- Math 6520 Introduction to Algebraic Topology
- Math 7800 Topics in Algebraic Geometry
- Reading Courses and Thesis Research in Algebraic Geometry

Mentoring

The chalkboard is the mathematics laboratory, and advising involves many hours of one-on-one interactions. I feel lucky to have had an extraordinary group of students and postdocs to mentor.

PhD Students Mentored

Henri Shahrouz. Harvard PhD 1995 (while I was a postdoc).

Thesis: *The Grothendieck Quot Scheme and Composition Laws*

Current Employment: Software Architect. NFL Enterprises.

Ionut Ciocan-Fontanine. Utah PhD 1996

Thesis: *The Quantum Cohomology Ring of Flag Varieties*

Current Employment: Professor. University of Minnesota.

Jian Kong. Utah PhD 2001

Thesis: Schubert Calculus on Flag Manifolds and Flag Bundles

Current Employment: Research Associate Scientist. Johns Hopkins.

Dragos Mustata. Utah PhD 2003

Thesis: Intermediate Moduli Spaces of Stable Maps

Current Employment: Lecturer. University College Cork, Ireland.

Renzo Cavalieri. Utah PhD 2005

Thesis: A Topological Quantum Field Theory for Admissible Covers

Current Employment: Associate Professor. Colorado State University.

Fumitoshi Sato. Utah PhD 2005

Thesis: Relations in the Tautological Ring Via Localization

Current Employment: Lecturer. Kagawa National College of Technology.

Gueorgui (Joro) Todorov. Utah PhD 2008

Thesis: Pluricanonical Maps for Threefolds

Current Employment: Quantitative Analyst. BlueCrest Capital.

Dylan Zwick. Utah PhD 2014

Thesis: Variations on a Theme of Symmetric Tropical Matrices

Current Employment: Overstock.com

Christian Martinez. Utah PhD 2015

Thesis: Stability Conditions and Birational Geometry

Current Employment: Assistant Professor (postdoc), UCSB

Drew Johnson. Utah PhD (expected) 2016

Thesis: Strange Duality and Tropical Geometry

Thomas Goller. Utah PhD (expected) 2017

Dapeng Mu. Utah PhD (expected) 2018.

Huachen Chen. Utah PhD (expected) 2018.

Franco Rota. Utah PhD (expected) 2019.

Postdocs Mentored

Holger Kley (1997-2000) Assistant Professor/Lecturer
Current Employment: Manager. Spirae.

Prakash Belkale (1999-2002) Wylie Assistant Professor/Lecturer
Current Employment: Professor. University of North Carolina.

Daniele Arcara (2003-2006) Assistant Professor/Lecturer
Current Employment: Chair and Associate Professor. Saint Vincent College.

Arend Bayer (2006-2008) Wylie Assistant Professor/Lecturer
Current Employment: Lecturer. University of Edinburgh.

Emanuele Macri (2008-2011) Wylie Assistant Professor/Lecturer
Current Employment: Assistant Professor. Ohio State University.

Steffen Marcus (2011-2014) Burgess Assistant Professor/Lecturer
Future Employment: Assistant Professor. The College of New Jersey.

Nicola Tarasca (2013-2016) Assistant Professor/Lecturer

Brooke Ullery (2015-) NSF Postdoctoral Scholar

Katrina Honigs (2015-) NSF Postdoctoral Scholar and Wylie Instructor

Undergraduates Mentored (Research Projects)

Drew Ellingson (2014-2015)

Project: Tropical Theta Characteristics

Hunter Simper (2015-2016)

Project: Tropical Plane Curves with Many Singularities

Dietrich Geisler (2015-2016)

Project: PCMI Lecture Notes

Teachers Mentored (Masters Projects):

Jessica Millar (2013)

Project: Quadratic Functions in Mathematics and Physics

Eric Schober (2014) Project: The Complex Circle

Chris Sheffield (2014) Project: Quadratic Reciprocity

Shinil Kang (2015)

Project: The Euclidean Solids

Kaitlyn Nordin and Izumi Okamura (2015)

Project: Ciphers and Cryptography

Lissett Sierra (2015)

Project: The Rubiks Cube

Vivian Shell (2015)

Project: The Real Projective Plane

4-5 more teacher projects in the works!

Recent Service

Mathematics Department Committees

2015-2016 Hiring Committee

2013-2015 Executive Committee

2012-2013 Chair of the Tenure-Track/Tenured Hiring Committee

2005-2011 Chair of the Mathematics Department.

2004-2005 Tenure-Track/Tenured Hiring Committee
Vision and Strategic Planning Committee
Outstanding Student and Instructor Awards
Mathematics Education Committee

2003-2004 Executive Committee
Graduate Admissions Committee
Mathematics Education Committee

2002-2003 Executive Committee
Graduate Recruitment and Admissions Committee
Mathematics Education Committee

College of Science Committees

2014-2018 College RPT Committee

2012-2013 Director, Math for America Utah

2011-2013 Center for Science and Math Education Steering Committee

2011-2012 George Thomas Building Educational Planning Committee

2003-2005 College of Science Council

University of Utah Committees

- University of Utah Teaching Committee
- University of Utah Research Committee
- Internal Reviewer for Graduate Programs

External Service

2009-2011 Member-at-Large, Council of the American Mathematical Society

2006-2012 Steering Committee for the Park City Mathematics Institute

2005-2007 Governor Huntsman's Math advisory board.