# A Very Incomplete Introduction to $IAT_EX$

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## 1 Warning!

This is only meant to provide an example of how to create a basic document in LATEX. You will undoubtably need more than this as a resource. There is a great resource called "The Not So Short Guide to LATEX" by Tobias Oetiker. You can find it online quite easily and download it.

## 2 Special Characters

There are special characters that are used within the .tex file. For example, the & symbol is used in many environments to separate sections, the % symbol is used to make comments in the .tex file, a \$ on each side of an expression puts that expression in math mode, the underscore is used for subscripts, braces  $\{,\}$  are used all over the place, and a  $\setminus$  is also used in virtually everything. Also, a double backslash  $\setminus$  will put a line break.

If you want to have these characters appear in the actual document, you have to use a backslash to do so. The special characters and the syntax for putting them in your document are given below.

& \& % \% # \# \$ \\$ - \-{ \{ } \} } \}

## 3 Environments

In this section, we will highlight a few of the most useful environments in LAT<sub>F</sub>X.

### 3.1 Math Mode

As mentioned, a dollar sign is used to typeset things in math mode. Without dollar signs, you get things looking like this: x-1. With dollar signs on either side, you get it to look like this: x - 1. If you want to have a mathematical expression centered and on a line of its own, put \$\$ on each side of the expression. For example,

$$\mathbb{Q} = \left\{ \frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0 \right\}$$

In any math environment, spaces in your .tex file will be ignored. If you need some extra horizontal space, you can use  $\langle ; \rangle$ :  $\langle , \rangle$ , or  $\langle hskip$ .

### 3.2 Align Environment

This is a math environment (everything is automatically in math mode) that allows you to align multiple lines. Here are a couple of examples.

$$\frac{dy}{dx} + \frac{2x}{x^2 + 1}y = \frac{1}{(x^2 + 1)^2} \tag{1}$$

$$(x^{2}+1)\frac{dy}{dx} + 2x \cdot y = \frac{1}{x^{2}+1}$$
(2)

$$(x^2 + 1)y = \int \frac{1}{x^2 + 1} dx \tag{3}$$

$$(x^2 + 1)y = \tan^{-1}x + C \tag{4}$$

$$y = \frac{\tan^{-1}x + C}{x^2 + 1} \tag{5}$$

$$d\omega = d(|x|^{-n}) \wedge \sum_{i=1}^{n} (-1)^{i-1} x_i \, dx_1 \wedge \dots \wedge \hat{dx_i} \wedge \dots \wedge dx_n$$
  
+  $|x|^{-n} \sum_{i=1}^{n} (-1)^{i-1} dx_i \wedge dx_1 \wedge \dots \wedge \hat{dx_i} \wedge \dots \wedge dx_n$   
=  $-n|x|^{-n-2} \left(\sum_{j=1}^{n} x_j dx_j\right) \wedge \sum_{i=1}^{n} (-1)^{i-1} x_i \, dx_1 \wedge \dots \wedge dx_i \wedge \dots \wedge dx_n$   
+  $n|x|^{-n} dx_1 \wedge \dots dx_n$   
=  $-n|x|^{-n-2} \sum_{i=1}^{n} (-1)^{i-1} x_i^2 \, dx_i \wedge dx_1 \wedge \dots \wedge \hat{dx_i} \wedge \dots \wedge dx_n$   
+  $n|x|^{-n} dx_1 \wedge \dots dx_n$   
=  $(-n|x|^{-n-2}|x|^2 + n|x|^{-n}) dx_1 \wedge \dots \wedge dx_n$   
=  $0,$ 

### 3.3 Tabular Environment

The tabular environment organizes items into a table. You can choose the number of rows and columns and, for every column, you can choose whether to center, left-justify, or right-justify the items in the column. You also have the option of inserting vertical and/or horizontal lines in your table. Here are some examples.

1st column	2nd column	3rd column	1st column	2nd column	3rd column
a	b	с	a	b	с
x	y	z	x	y	z

1st column	2nd column	3rd column	1st column	2nd column	3rd column
a	b	с	a	b	С
x	y	z	x	y	z

### 3.4 Array Environment

This environment can be used to typeset matrices. It works practically the same as the tabular environment, but it is a math environment. Here are a couple of examples.

$$SL(2,\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{R}) : ad - bc = 1 \right\}$$
$$\begin{pmatrix} \frac{a_{11} \mid a_{12} \cdots a_{1n}}{0 \mid 1 \cdots \mid 0} \\ \vdots \quad \vdots \quad \ddots \quad \vdots \\ 0 \mid 0 \cdots \mid 1 \end{pmatrix}$$

### 3.5 Itemize and Enumerate Environments

The itemize and enumerate environments are used to create lists. The itemize environment should be used when the items in the list do not need to be "numbered."

- The default setting for itemize gives bullets.
- ! You can substitute another symbol at a specific stage by typing \item[insert symbol].
- + You can also substitute another symbol for the whole list. This is done before the list begins with the command \begin{itemize}[insert symbol].

The enumerate environment should be used when you want items of the list ordered.

- 1. You can choose what symbols you want to order your items. After \begin{enumerate}, you can put 1, a, i, or  $\alpha$ , in brackets.
- 2. For this list, [1.] was inserted.
- 3. You can have other symbols besides periods, e.g. [(i)] or [a:].

### 3.6 Cases Environment

This environment is nice for defining piecewise functions, denoting simultaneous equations, etc. You can align one spot, using &. For example,

$$|x| = \begin{cases} x & \text{if } x \ge 0\\ -x & \text{if } x < 0 \end{cases}$$

Another example:

$$\begin{cases} x'' + 4x' - 5 = 0\\ x(0) = 0\\ x'(0) = 6 \end{cases}$$

### 3.7 Figure Environment

To include a figure, you can use the graphicx package or the epsfig package. Included below is the same figure twice, one with each package.



Also, you can put these in a figure environment if you would like to include a caption. This also gives you more control of where the figure appears in the document.



Figure 1: Codewheel representation of the 3-bit reflected Gray code

## 4 Theorem Styles

This section will contain some (disconnected) examples of definitions, theorems, etc, pulled from various notes.

**Definition 4.1.** For an integer  $n \ge 0$ ,  $\zeta \in \mathbb{C}$  is called an  $n^{th}$  root of unity if  $\zeta^n = 1$ . If there is no integer  $d, 1 \le d < n$ , such that  $\zeta^d = 1$ , then  $\zeta$  is called a *primitive*  $n^{th}$  root of unity.

Example 4.2. Let  $f \in \mathbb{Q}[x]$  be an irreducible polynomial of degree 5 with exactly two non-real roots. Let  $\mathbb{E}$  be the splitting field of f over  $\mathbb{Q}$ . Let  $\alpha \in \mathbb{E}$  be a root of f. Then  $[\mathbb{Q}(\alpha) : \mathbb{Q}] = 5$ , giving that 5 divides |G| where  $G = \operatorname{Gal}(\mathbb{E}/\mathbb{Q})$ . Hence, if G is viewed as a subgroup of  $S_5$ , then Gcontains a 5-cycle. Let  $\sigma : \mathbb{C} \to \mathbb{C}$  be given by  $\sigma(z) = \overline{z}$ . Then  $\sigma \in \operatorname{Aut}(\mathbb{C})$  and  $\sigma|_{\mathbb{R}}$  is the identity, giving that  $\sigma \in \operatorname{Gal}(\mathbb{C}/\mathbb{R})$ . Thus  $\sigma \in G$ . Since  $\sigma^2$  is the identity and  $\sigma$  is not, then G contains a transposition. Therefore  $G = S_5$  as G contains both a transposition and a 5-cycle. Hence, f is not solvable by radicals over  $\mathbb{Q}$  as  $G = S_5$  is not solvable.

**Proposition 4.3.** Suppose a Riemannian manifold M has an affine connection  $\nabla$  compatible with  $\langle, \rangle$ . Let V, W be vector fields along a  $c : I \to M$ . Then

$$\frac{d}{dt}\left\langle V,W\right\rangle = \left\langle \frac{DV}{dt},W\right\rangle + \left\langle V,\frac{DW}{dt}\right\rangle.$$

*Proof.* Let  $P_1(t), \ldots, P_n(t)$  be an orthonormal frame of parallel vectors along c. Such a frame exists as one can take the parallel transport of an orthonormal basis of  $T_{c(t_0)}M$ . Then V and W can be written as

$$V = \sum_{i=1}^{n} v_i(t) P_i(t),$$
$$W = \sum_{j=1}^{n} w_j(t) P_j(t).$$

Since  $P_i$  is parallel along c, then  $\frac{DP_i}{dt} = 0$  and hence

$$\frac{DV}{dt} = \sum_{i=1}^{n} \frac{dv_i}{dt} P_i(t),$$
$$\frac{DW}{dt} = \sum_{j=1}^{n} \frac{dw_j}{dt} P_j(t).$$

Therefore,

$$\left\langle \frac{DV}{dt}, W \right\rangle + \left\langle V, \frac{DW}{dt} \right\rangle = \sum_{i=1}^{n} \left( \frac{dv_i}{dt} w_i + v_i \frac{dw_i}{dt} \right)$$
$$= \frac{d}{dt} \left( \sum_{i=1}^{n} v_i w_i \right)$$
$$= \frac{d}{dt} \left\langle V, W \right\rangle.$$

**Theorem 4.4.** Let  $\mathbb{F}_q$  be a finite field and let C be a BCH code with parameters n, d. Then  $d(C) \geq d$ .

*Proof.* Let r be such that  $q^r > n$ , let f be an irreducible polynomial in  $\mathbb{F}_q[x]$  with  $\deg(f) = r$ , and let  $\alpha$  be any primitive element of the finite field  $\mathbb{F} = \mathbb{F}_q[x]/(f)$ . Define  $g(x) = \prod m_i(x)$  where the product is take over distinct terms and  $m_i(x)$  is the product of terms of the form  $x - \gamma$  where  $\gamma$  is in the conjugacy class of  $\alpha^i$ . Then

$$C = \left\{ f(x)g(x) : f \in \mathbb{F}_q[x], \deg(f) < n - \deg(g) \right\}$$

is a BCH code with parameters n, d. Since C is a polynomial code, it is also a group code, and hence  $d(C) = \min\{\operatorname{wt}(c) : c \neq 0, c \in C\}$ . For  $c \in C$  nonzero, there exists  $a \in \mathbb{F}_q[x]$  such that c(x) = a(x)g(x). Therefore we have

- (i)  $c(\alpha^i) = 0$  for  $i = 1, 2, \dots, d-1$ ,
- (ii)  $\deg(c) < n \le q^r 1 = \operatorname{order}(\alpha)$ .

Since  $c \neq 0$ , then the number of nonzero coefficients of c is at least d-1, giving that  $wt(c) \geq d$ .  $\Box$ 

To reference a previous theorem, definition, etc, label an item with \label and refer to it using \ref. For Theorem 4.4, we used the label {bch}, so to refer to it, we type  $\{bch\}$ . You need to compile the document a couple of times to get the references to show up correctly. Using \ref and \label is better than labeling things yourself, as LATEX will keep track of the numbering for you.

**Lemma 4.5** (Second Bianchi Identity).  $R_i{}^j{}_{k\ell,m} + R_i{}^j{}_{\ell m,k} + R_i{}^j{}_{mk,\ell} = 0.$ 

*Proof.* Recall the defining relation  $d\omega_i{}^j - \omega_i{}^k\omega_k{}^j = -\frac{1}{2}R_i{}^j{}_{k\ell} \omega^k \wedge \omega^\ell$ . If we differentiate this relation, we obtain

$$-(d\omega_i k) \wedge \omega_k{}^j + \omega_i{}^k \wedge (d\omega_k{}^j) = -\frac{1}{2} R_i{}^j{}_{k\ell} \Big( (d\omega^k) \wedge \omega^\ell - \omega^k \wedge (d\omega^\ell) \Big).$$

Using the structure equations

$$\begin{cases} \omega_i{}^j + \omega_j{}^i &= 0\\ \omega^j \wedge \omega_j{}^i &= d\omega^i \end{cases}$$

The differentiated relation simplifies to

$$0 = \frac{1}{2} \Big[ dR_i{}^j{}_{pq} + R_i{}^k{}_{pq} \omega_k{}^j - R_k{}^j{}_{pq} \omega_i{}^k - R_i{}^j{}_{kq} \omega_p{}^k - R_i{}^j{}_{p\ell} \omega_q{}^\ell \Big] \omega_p \wedge \omega^q$$
  
$$= \frac{1}{2} R_i{}^j{}_{pq,k} \omega^k \wedge \omega^p \wedge \omega^q$$
  
$$= R_i{}^j{}_{pk,q} + R_i{}^j{}_{kq,p} + R_i{}^j{}_{qp,k}$$

**Hopf-Rinow Theorem.** If M is a connected Riemannian manifold, then the following are equivalent.

- (a) For some  $p \in M$ , every geodesic through p is defined for all t.
- (b) Closed and bounded subsets of M are compact.
- (c) M is complete as a metric space.
- (d) M is geodesically complete.

Moreover, each of (a) through (d) implies that for every  $p, q \in M$ , there exists a geodesic in M connecting p and q of length d(p,q).

*Proof.* The proof is left as an exercise.