# Summer High School Math 2009
# Some Data

## Perfect Squares Modulo Some Primes

| | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | 121 | 144 | 169 | 196 | 225 | 256 | 289 | 324 | 361 | 400 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 1 | 1 | 0 | | | | | | | | | | | | | | | | | |
| 5 | 1 | -1 | -1 | 1 | 0 | | | | | | | | | | | | | | | |
| 7 | 1 | -3 | 2 | 2 | -3 | 1 | 0 | | | | | | | | | | | | | |
| 11 | 1 | 4 | -2 | 5 | 3 | 3 | 5 | -2 | 4 | 1 | 0 | | | | | | | | | |
| 13 | 1 | 4 | -4 | 3 | -1 | -3 | -3 | -1 | 3 | -2 | 4 | 1 | 0 | | | | | | | |
| 17 | 1 | 4 | -8 | -1 | 8 | 2 | -2 | -4 | -4 | -2 | 2 | 8 | -1 | -8 | 4 | 1 | 0 | | | |
| 19 | 1 | 4 | 9 | -3 | 6 | -2 | -8 | 7 | 5 | 5 | 7 | -8 | -2 | 6 | -3 | 9 | 4 | 1 | 0 | |
| 23 | 1 | 4 | 9 | -7 | 2 | -10 | 3 | -5 | 12 | 8 | 6 | | | | | | | | | |
| 29 | 1 | 4 | 9 | -13 | -4 | 7 | -9 | 6 | -6 | 13 | 5 | -1 | -5 | -7 | | | | | | |
| 31 | 1 | 4 | 9 | -15 | -6 | 5 | -13 | 2 | -12 | 7 | -3 | -11 | 14 | 10 | 8 | | | | | |
| 37 | 1 | 4 | 9 | 16 | -12 | -1 | 12 | -10 | 7 | -11 | 10 | -4 | -16 | 11 | 3 | -3 | -7 | -9 | | |
| 41 | 1 | 4 | 9 | 16 | -16 | -5 | 8 | -18 | -1 | 18 | -2 | -20 | 5 | -9 | 20 | 10 | 2 | -4 | -8 | -10 |

**Notice:**

- $-1$ is a square modulo: $5, 13, 17, 29, 37, 41$ but not modulo: $3, 7, 11, 19, 23, 31$.
- 2 is a square modulo: $7, 17, 23, 31, 41$ but not modulo: $3, 5, 11, 13, 19, 29, 37$.

## Powers of a Primitive Modulo Some Primes

(starred numbers are the other primitives)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 2* | 1 | | | | | | | | | | | | | | | | | | |
| 5 | 2* | -1 | $-2^*$ | 1 | | | | | | | | | | | | | | | | |
| 7 | 3* | 2 | -1 | -3 | $-2^*$ | 1 | | | | | | | | | | | | | | |
| 11 | 2* | 4 | $-3^*$ | 5 | -1 | -2 | $-4^*$ | 3 | $-5^*$ | 1 | | | | | | | | | | |
| 13 | 2* | 4 | -5 | 3 | 6* | -1 | $-2^*$ | -4 | 5 | -3 | $-6^*$ | 1 | | | | | | | | |
| 17 | 3* | -8 | $-7^*$ | -4 | 5* | -2 | $-6^*$ | -1 | $-3^*$ | 8 | 7* | 4 | $-5^*$ | 2 | 6* | 1 | | | | |
| 19 | 2* | 4 | 8 | -3 | $-6^*$ | 7 | $-5^*$ | 9 | -1 | -2 | $-4^*$ | -8 | 3* | 6 | -7 | 5 | $-9^*$ | 1 | | |
| 23 | 5* | 2 | 10* | 4 | $-3^*$ | 8 | $-6^*$ | -7 | 11* | 9 | -1 | | | | | | | | | |
| 29 | 2* | 4 | 8* | -13 | 3* | 6 | 12 | -5 | $-10^*$ | 9 | $-11^*$ | 7 | 14* | -1 | | | | | | |
| 31 | 3* | 9 | -4 | -12 | -5 | -15 | $-14^*$ | -11 | -2 | -6 | 13* | 8 | $-7^*$ | 10 | -1 | | | | | |
| 37 | 2* | 4 | 8 | 16 | $-5^*$ | -10 | 17* | -3 | -6 | -12 | 13* | -11 | 15* | -7 | -14 | 9 | 18* | -1 | | |
| 41 | 7* | 8 | 15* | -18 | -3 | 20 | 17* | -4 | 13* | 9 | $-19^*$ | -10 | 12* | 2 | 14 | 16 | $-11^*$ | 5 | $-6^*$ | -1 |