Math 7800/Fall 2019
Fano Varieties and Calabi-Yau Categories
M,W 1:25-2:25, LCB 323 (the schedule is negotiable)

The stability conditions and moduli of complexes of vector bundles (D-branes) give rise to invariants of Calabi-Yau and Fano varieties with applications to algebraic geometry and mirror symmetry. The main purpose of this course is to explore four examples: K3 surfaces, Fano and quintic threefolds and cubic fourfolds. We will also investigate the Homological Projective Duality conjecture of Kuznetsov since Kuznetsov components are "non-commutative" Calabi-Yau categories associated to Fano varieties supporting stability conditions.

This is an advanced seminar course. One year of graduate algebraic geometry (e.g. Harshorne, Algebraic Geometry) is a must, and one more year is desirable. The course will involve student participation, since the goal is for the students, and not just the instructor, to be in a position to frame and work on open problems in the area.

Topics to be covered with references (in this order).

Triangulated Categories and Stability Conditions.

[0] Huybrechts, Fourier-Mukai Transforms in Algebraic Geometry.

[1] Lectures on Bridgeland stability, arXiv:1607.01262.

Homological Projective Duality

[2] Notes on HPD, arXiv:1512.08985.

K3 Surfaces

[3] MMP for moduli spaces of sheaves on K3 surfaces, arXiv:1301.6968

Fano Threefolds

[4] Bridgeland stability conditions on Fano threefolds, arXiv:1607.08199

The Quintic Threefold

[5] On stability conditions for the quintic threefold, arXiv:1810.03434.

The Cubic Fourfold

[6] Stability conditions on Kuznetsov components, arXiv:1703.10839.

The first class on Monday, August 19 will be mostly organizational. The motivated student may look at [0], [1] and [2] before the start of class and assist with the first several weeks of lectures (every student will eventually assist with the lectures). Aaron Bertram.