**Math 6140/Algebraic Geometry/Spring 2017**
**Syllabus**

Course webpage: www.math.utah.edu/~bertram/6140

Class meets: MWF 11:50-12:40 in LCB 222

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Office Hours: Before class (11-11:50 MWF)

**Text:** Course Notes and Hartshorne, *Algebraic Geometry*

**Grading:** Grades will be based on homework.

**Where we are:** A **variety** over an algebraically closed field $k$ is a Noetherian topological space $X$ together with a sheaf $\mathcal{O}_X$ of $k$-algebras that is *locally affine*, i.e. covered by open sets obtained from the mspec construction, and *separated*. The mproj construction obtains a projective (proper) variety from a finitely generated (in degree one) *graded* $k$-algebra domain $R_\bullet$, with $R_0 = k$. Unlike the case with mspec, non-isomorphic graded rings may yield isomorphic projective varieties.

In the Fall we explored the mspec and mproj constructions, their functoriality (or lack thereof) and we investigated **morphisms** between varieties, including open and closed embeddings and dominant and finite maps. We also enlarged the concept of a morphism to include *rational maps*, which were very useful, for example, in characterizing a morphism $f : X \to \mathbb{P}^n$ to projective space. We also discussed *local* properties of varieties, obtained from the **local rings** $\mathcal{O}_{X,x}$ attached to points $x \in X$. A variety $X$ was defined to be **normal** at $x \in X$ if $\mathcal{O}_{X,x}$ is integrally closed and **nonsingular** if $\mathcal{O}_{X,x}$ is regular. We proved that every variety $X$ has a **normalization**, i.e. a uniquely determined finite, surjective map $\nu : \widetilde{X} \to X$ from a normal variety, which is projective when $X$ is projective.

We also discussed Weil and Cartier divisors and their **class groups**, which are isomorphic when $X$ is nonsingular. This led to another characterization of a (rational) map $f : X - - > \mathbb{P}^n$ in terms of linear series. Finally, we began our discussion of coherent sheaves, introducing line bundles and vector bundles and in particular we interpreted a linear series as a vector space of sections of a line bundle on $X$, allowing for a third way of describing a (rational) map to projective space.

**Where we are going.** Coherent sheaves are sheaves of $\mathcal{O}_X$ modules on a variety that are locally equivalent to finitely generated $A$-modules over mspec($A$). Important examples include vector bundles (locally free sheaves), ideal sheaves, and the sheaf of differentials $\Omega_X$, which is locally free in the case that $X$ is non-singular. This latter sheaf is canonically associated to a variety and can therefore be used to define numerical invariants of a variety (for example, the genus of a nonsingular curve). We will explore the abelian category of coherent sheaves on $X$, and functors between categories of coherent sheaves on $X$ and $Y$ induced from a morphism $f : X \to Y$ of varieties. We will also see that multi-linear algebra constructions can be applied to coherent sheaves, and obtain the **dualizing sheaf** $\omega_X$ of a non-singular variety, defined as the top exterior power of the vector bundle of differentials. Coherent sheaves also allow for some *relative constructions*, including the bundle of projective spaces over $X$ associated to a vector bundle on $X$ and the **blow-up** of $X$ along a closed subvariety.

We will turn next to the **cohomology** of coherent sheaves, defined via Cech complexes associated to open covers of $X$. The cohomology captures *global* information of a coherent sheaf in terms of the patching of local information. It also highlights the special role of **ample** line bundles on a (projective) variety $X$ via Serre's Theorems A and B. The dualizing sheaf $\omega_X$ then earns its name with its crucial role in the **Serre Duality Theorem** which can be thought of as an analogue of duality theorems in algebraic topology.

Time permitting, we will apply this machinery to the study of curves and surfaces, the latter requiring some **intersection theory**, specifically the intersection number of a pair of curves on a projective surface. We will discuss Riemann-Roch theorems, adjunction formulas, Hodge index theorem and a bit of the classification of curves and surfaces.

**ADA Statement:** The Americans with Disabilities Act requires that reasonable accomodations be provided for each student with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss whether such accommodations are necessary.