Math 4800/6080. Week Six Starter

1. Show that every line:

\[(ax + by + cz = 0)\]

in \(\mathbb{RP}^2\) is the image of a 1-1 “linear” map from \(\mathbb{RP}^1\) to \(\mathbb{RP}^2\) of the form:

\[\Phi(u : v) = (s_0u + t_0v : s_1u + t_1v : s_2u + t_2v)\]

for some constants \(s_0, t_0, s_1, t_1, s_2, t_2\).

2. Show that the standard unit circle, hyperbola and parabola:

\[(x^2 + y^2 - z^2 = 0), \ (x^2 - y^2 - z^2 = 0)\text{ and } (yz - x^2 = 0)\]

are all images of 1-1 “quadratic” maps from \(\mathbb{RP}^2\) to \(\mathbb{RP}^2\) of the form:

\[\Psi(u : v) = (q_0(u, v) : q_1(u, v) : q_2(u, v))\]

where \(q_0, q_1, q_2\) are homogeneous quadratic polynomials.

3. Play with triples:

\[(c_0(u, v) : c_1(u, v) : c_2(u, v))\]

of homogeneous cubic polynomials in \(u, v\) and see what the image looks like. Is the map from \(\mathbb{RP}^1\) to \(\mathbb{RP}^2\) ever 1-1 onto its image?