Math 4800/6080. Week Six Starter

1. Show that every line:

$$(ax + by + cz = 0)$$

in $\mathbb{RP}^2$ is the image of a 1-1 "linear" map from $\mathbb{RP}^1$ to $\mathbb{RP}^2$ of the form:

$$\Phi(u : v) = (s_0 u + t_0 v : s_1 u + t_1 v : s_2 u + t_2 v)$$

for some constants $s_0, t_0, s_1, t_1, s_2, t_2$.

2. Show that the standard unit circle, hyperbola and parabola:

$$(x^2 + y^2 - z^2 = 0),\ (x^2 - y^2 - z^2 = 0) \text{ and } (yz - x^2 = 0)$$

are all images of 1-1 "quadratic" maps from $\mathbb{RP}^2$ to $\mathbb{RP}^2$ of the form:

$$\Psi(u : v) = (q_0(u, v) : q_1(u, v) : q_2(u, v))$$

where q_0, q_1, q_2 are homogeneous quadratic polynomials.

3. Play with triples:

$$(c_0(u, v) : c_1(u, v) : c_2(u, v))$$

of homogeneous cubic polynomials in u, v and see what the image looks like. Is the map from $\mathbb{RP}^1$ to $\mathbb{RP}^2$ ever 1-1 onto its image?