Math 4800/6080. Week Ten Starter

1. Let $r_1, r_2$ be the two roots (maybe complex) of the polynomial:

   $$ax^2 + bx + c$$

   (a) Find expressions for $b$ and $c$ in terms of the two roots and $a$.
   
   (b) Express the discriminant:

   $$b^2 - 4ac$$

   in terms of the two roots.

2. Let $r_1, r_2, r_3$ be the three roots (maybe complex) of the polynomial:

   $$ax^3 + bx^2 + cx + d$$

   Find expressions for $b, c, d$ in terms of the three roots.

3. Generalize 1(a) and 2 to polynomials of any degree.

4. Use Sylvester’s determinant to express the discriminants of:

   (a) $x^3 + px + q$
   
   (b) $x^3 + rx^2 + s$
   
   (c) $x^4 + px + q$
   
   (d) $x^4 + rx^2 + s$
   
   (e) $x^4 + tx^3 + u$