Math 4800/6080. Week Two Starter Problem

1. Given a point \((x_1, y_1)\) in the plane, find a translation such that:
   \[
   T \left( \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
   \]

2. Given two points \((x_1, y_1)\) and \((x_2, y_2)\), find a similarity such that:
   \[
   T \left( \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
   \]
   and
   \[
   T \left( \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
   \]

3. Suppose the similarity you find in Problem 2. is a congruence. What can you conclude about the two points?

4. Given three points \((x_1, y_1)\), \((x_2, y_2)\) and \((x_3, y_3)\) that are not collinear, find an affine linear transformation such that:
   \[
   T \left( \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
   \]
   and
   \[
   T \left( \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
   \]
   and
   \[
   T \left( \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
   \]

5. Suppose the transformation you find in Problem 4. reverses the orientation of the plane. What can you conclude about the triangle in \(\mathbb{R}^2\) with vertices \((x_1, y_1)\), \((x_2, y_2)\) and \((x_3, y_3)\)?