Definitions you need to know.

1. What is a ring?
2. What is field?
3. What is the characteristic of a field?
4. What is a quadratic integer?
5. What is a perfect number?
6. What is a Mersenne prime?
7. What is the function $\sigma(n)$?
8. What is a primitive $d$th root of 1 in a field $F$?
9. What is the discrete logarithm $I_a$ in $(\mathbb{Z}/p\mathbb{Z})^\times$?
10. What is the cyclotomic polynomial $\Phi_d(x)$?

How to’s.

1. How to solve linear equations mod $p$.
2. How to find $m$th roots (mod $p$) when gcd($m, p - 1$) = 1.
3. How to find multiplicative inverses in $\mathbb{Q}[\sqrt{d}]$?
4. How to find multiplicative inverses in $\mathbb{Z}[\sqrt{d}]$ (mod $p$).
5. How to compute $\sigma(n)$.
6. How to find high powers $a^u$ (mod $p$) by successive squaring.

Theorems you should be able to state precisely.

1. Wilson’s Theorem
2. The Lucas-Lehmer Test
3. Dirichlet’s Theorem on Primes in Arithmetic Progressions

Stuff you should be able to prove.

1. Wilson’s Theorem.
2. Every even perfect number is of the form $2^{l-1}(2^l - 1)$ where $2^l - 1$ is a Mersenne prime.
3. $\mathbb{Q}[\sqrt{d}]$ is a field whenever $d \in \mathbb{Z}$ and $\sqrt{d}$ is irrational.
4. $\sum_{d|n} \phi(d) = n$. 