Calculus with a Complex Variable

In ordinary calculus, a function $f(t)$ is:

continuous if its graph can be drawn without lifting the pen

differentiable if its graph can be drawn without stopping the pen

We can use "i" to define a **complex** variable:

$$z = s + it$$

and it is **much** harder for a function $f(z)$ to be differentiable. Specifically, $f(z)$ is actually a pair of functions:

$$f = u + iv$$

that are required to be **harmonic**, meaning that:

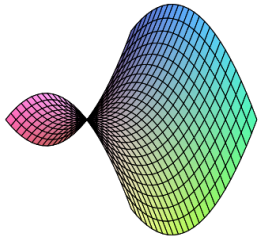
$$\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2} = 0 \quad \text{and} \quad \frac{\partial^2 v}{\partial s^2} + \frac{\partial^2 v}{\partial t^2} = 0$$

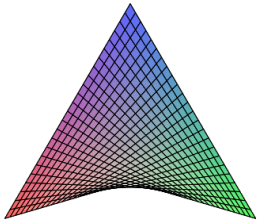
(for the experts). Such functions are rare and beautiful.

Examples of Differentiable Complex Functions

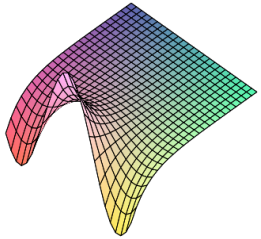
Examples (Functions $f(z)$ followed by the graphs of u and v)

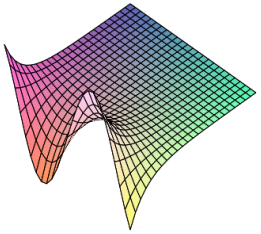
$f(z) = z^2$





$f(z) = e^z$





$f(z) = \ln(z)$