Math 4200-001/Complex Analysis/Fall 2017
First Midterm

1. (10 points) Compute each of the following in the form $a + bi$ by converting to polar, taking the power and then converting back:

(a) $(\sqrt{3} + i)^8$ (b) $\left(\frac{1}{2} + \frac{1}{2}i\right)^8$

2. (10 points) What are the five 5th roots of $-1 - i$? (in polar form).

3. (10 points) For a complex number $a + bi$, use the principal value of $\log(z)$ to define:

$$z^{a+bi} = e^{(a+bi)\log(z)}$$

Show that z^{a+bi} extends to a continuous function across the negative real axis if and only if a is an integer and $b = 0$.

4. (15 points) Find power series $\sum a_n z^n$ satisfying each of the following:

(a) $\sum a_n z^n$ has radius of convergence 1 and converges at every point of the unit circle.

(b) $\sum a_n z^n$ has radius of convergence 1 and converges at **no** point of the unit circle.

(c) $\sum a_n z^n$ has radius of convergence 1 and converges at 1 and -1 but not at i and not at $-i$.

5. (10 points) Prove that each half plane $\theta < \arg(z) < \theta + \pi$ is an open subset of $\mathbb{C}$, and use this to prove that each sector $\theta < \arg(z) < \psi$ is an open subset of $\mathbb{C}$. Hint: You may use the fact that multiplication by any constant is a continuous function.

6. (20 points) Suppose $f(z) = f(x + iy)$ is a differentiable function. Which of the following functions are necessarily also differentiable? (Justify your answers!)

(a) $f(1/z)$ (b) $f(iz)$ (c) $\overline{f(\overline{z})}$ (d) $\overline{f(z)} \cdot f(\overline{z})$

7. (15 points) For each of the following harmonic functions $u(x, y)$, find a *conjugate* harmonic function $v(x, y)$ so that $f(z) = u(x, y) + iv(x, y)$ is differentiable.

(a) $u(x, y) = xy$, (b) $u(x, y) = \log(x^2+y^2)$, (c) $u(x, y) = \cot^{-1}(x/y)$

8. (10 points) Show that if $f(z) = u + iv$ is differentiable, then:

$$\log|f(z)| = \log\left(\sqrt{u^2 + v^2}\right) \text{ is harmonic}$$