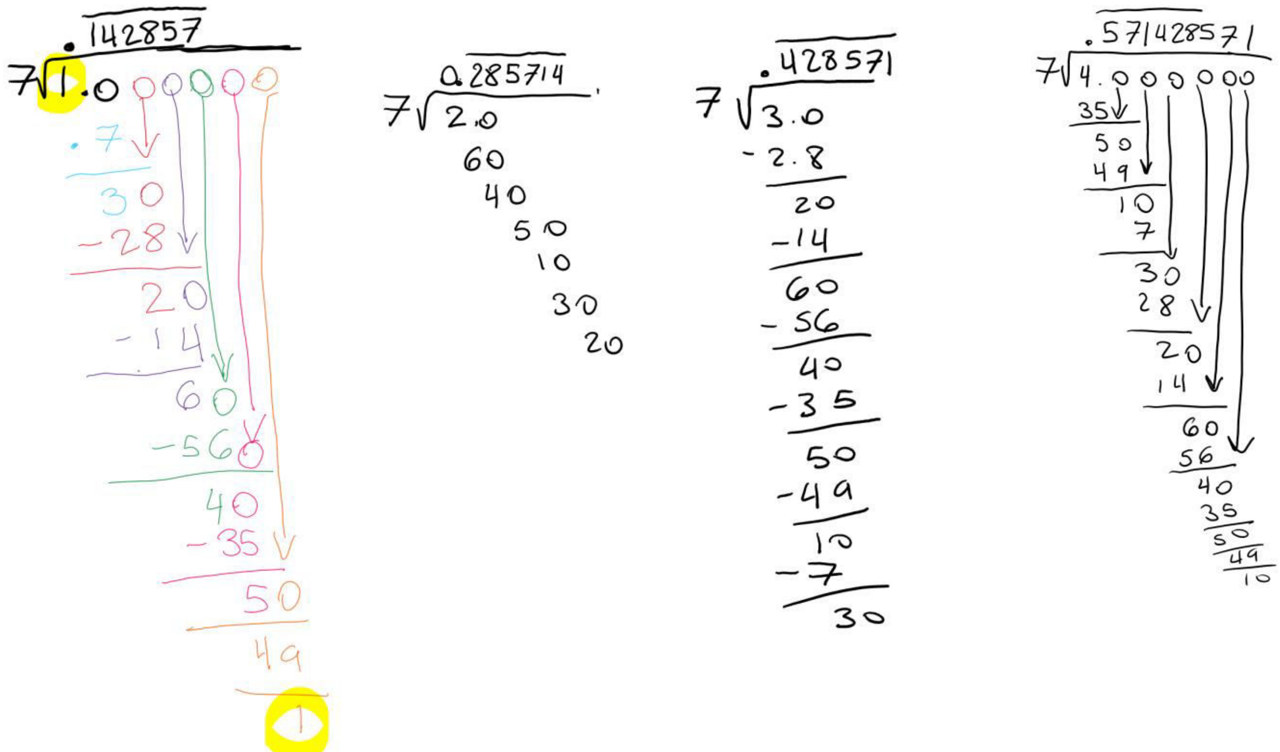
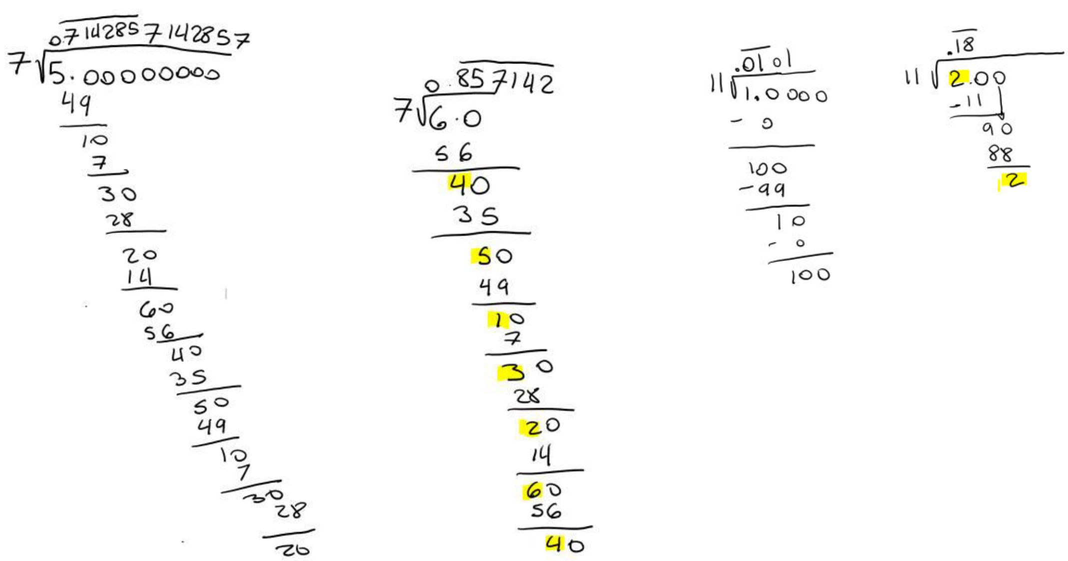
Infinite decimal expansions of rational number

Terminating decimal- if you multiply it by a multiple of ten you get a whole number
Repeating decimal - is a decimal number with a digit, or group of digits, that repeat on and on, without end
Every fraction repeats





Notice that we have repeating decimals made up of the same digits. When you do 1/7 you're going through every possible remainder and shifting the decimal result according to the first remainder.

1/11=.0909
2/11=.1818
There are only two possible remainders when going through long division in either case so the repeating decimal has a repeating block of two digits.

<u>All rational numbers will repeat!</u>
Observe the rational number 5/13. Because it can be written in the form p/q, where q does not equal zero, we know this will repeat eventually
5/13=0.3846153
6/13=0.461538
When we increase the numerator and keep the same denominator, the repeating decimal will shift over beginning with that remainder

$$\phi(14) = \{1, \cancel{2}, 3, \cancel{4}, 5, \cancel{6}, \cancel{7}, \cancel{8}, 9, \cancel{10}, 11, \cancel{12}, 13\}$$

$$\phi(2\cdot 7) = \underset{\underset{1}{\parallel}}{\phi(2)} \cdot \underset{\underset{6}{\parallel}}{\phi(7)} = 6$$

$$\phi(27) = \phi(3^3) = 3^3 - 3^2 = 18$$

<u>Euler's theorem to find the length of a repeating block</u>
If we have a number n
$\phi$(n)= number of remainders that are relatively prime to n

These wil satisfy gcd(r,n)=1
The number of relatively prime numbers will be the repeating block length
$\phi$(14)={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}, here there are 6 numbers relatively prime to 14 so the repeating block of a fraction with denominator 14 will be 6 digits long

<u>Fermat's little theorem</u> says the length of the repeating block will be $10^{p-1}$ when $p$ is a prime
When p=7, repeat after 6 places
p=11, repeat after 10 places

$0.\overline{384615}3$

$13 \overline{)\,500000\ldots}$

39
110
104
60
52
80
78
20
13
70
65
50

$10 \equiv_{13} -3 = 10$

$10^2 \equiv 9 \equiv (-3)^2$

$10^3 = 9(-3) = -27 \equiv_{13} -1 \equiv_{13} 12$

$10^4 \equiv_{13} 9^2 = 3$

$10^5 \equiv_{13} 3 \cdot -3 = -9 \equiv_{13} 4$

$10^6 \equiv -9 \cdot -3 \equiv_{13} 1$

$10_7 \equiv_{13} 10$

$0.769230$

$13 \overline{)\,10}$

91
90
78
120
117
30
26
40
39
0

$100 = 7 \cdot 13 + 9$

$1000 = 76 \cdot 13 + 12$

ect.