Different Bases: Wednesday, September 21st

Background:

- The most commonly used base in the United States school system is base 10. The digits used in this base are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. When we increase by one digit more, we write "10" which means "one ten and zero ones". Base 10 is widely used because we have ten fingers, making it easier to count using our fingers.
- The numbers used in each base start at 0 and end at n-1 with n being the base. For example in base 5 we use digits 0, 1, 2, 3, 4.

If base 10 is relatively easy for us to use, why do we use other bases?

- Computers commonly use bases such as base 2 and base 16.
- As future teachers, learning about different bases is a good way to grasp how students feel when first performing arithmetic in base 10. Learning how to carry out multiplication, addition, subtraction, etc. in different bases can be difficult at first. Students may feel this way when they learn arithmetic in base 10. Knowing the frustrations our students may experience can help us change our teaching practices to become better educators.

Learning Activity: Grouping with Straws

To help students understand what we mean by "base 10," have students work in small groups to count a bucket filled with straws. When we were asked to do this as a class, we instinctively put the straws in groups of ten. Then we gathered ten groups of ten to make a group of one hundred. Using this method made the process of counting the straws relatively fast and easy.

We were then asked to count the straws again, but this time by making our smallest group be a group of five straws. Then the next smallest group would be a group of twenty-five straws or five groups of five straws.

In doing this activity, we learned that the key to understanding different bases is knowing what your smallest group size is.

Examples of Arithmetic in Different Bases

1. Convert 994_{10} to base 5.
   Base 5 uses groups of 5, 25, 125, 625, and so on. The idea is to subtract off multiples of these groups from 994 until we have no more remainders.

   $994 - (1)625 = 369$, so we have 1 group of $625 = 5^4$.
   $369 - (2)125 = 119$, we have 2 groups of $125 = 5^3$.
   $119 - (4)25 = 19$, we have 4 groups of $25 = 5^2$.
   $19 - (3)5 = 4$, we have 3 groups of $5 = 5^1$
   $4 - (4)1 = 0$, we have 4 groups of $1 = 5^0$.

So, $994_{10} = 12434_5$.

2. Add 1340_5 and 312_5.
   We will set up this addition as we would when working in base 10.
   - Add *0 groups of 1* and *2 groups of 1*. This makes *2 groups of 1*.

     1340
     + 312
     ——————
     2
   - Add *4 groups of 5* and *1 group of 5*. This makes *5 groups of 5* which is *1 group of 25*, and we write this as *10*, and we carry the *1* to the next column (shown in blue).

     1
     1340
     + 312
     ——————
     02
   - Add *1 group of 25*, *3 groups of 35*, and *3 groups of 25*. This makes *7 groups of 25*, or *1 group of 125* and *2 groups of 25*. We carry the 1 from the group of 125 (shown in blue).

     11
     1340
     + 312
     ——————
     202
   - Add *1 group of 125* and *1 group of 125*. This makes *2 groups of 125*.

     11
     1340
     + 312
     ——————
     2202
   - So, $1340_5 + 312_5 = 2202_5$

3. Subtract 132_5 from 431_5.
   - Similar to the example above, we will be using groups of 5, 25, 125,....
   - We need to subtract 2 groups of 1 from 1 group of 1, so we will pull a group of 5 from the column to the left and add it to the 1 group of 1, so we have 6 groups of 1 in total. 6 minus 2 is 4 groups of 1.

     26
     ~~431~~
     - 132
     ——————
     4
   - Now we have 2 groups of 5 (since we pulled a group of 5 out in the previous step) and we want to subtract 3 groups of 5. Pull a group of 25 from the column to the left and add it to our 2 groups of 5. Now we have 7 groups of 5 minus 3 groups of five, which leaves 4 groups of 5.

7
3~~2~~6
~~4~~31
- 1~~3~~2
4~~4~~

- We have 3 groups of 25 in the leftmost column minus 1 group of 25. This leaves 2 groups of 25.

7
3~~2~~6
~~4~~31
- 1~~3~~2
2~~4~~4

- So, $431_5 - 132_5 = 244_5$.