

For  $n^x$  where the base  $n$  is a natural number, the exponent  $x$  tells us how many times  $n$  will be multiplied by itself.

A function that grows exponentially grows at an increasing slope and goes towards infinity faster than any other polynomial function.  $y = e^x$

The base of an exponential determines whether the number will increase or decrease

When base  $0 < b < 1$  it decreases

When  $b > 1$  it increases

### Exponential rules

Product  $a^x * a^y = a^{x+y}$

$$a^x * b^x = (a * b)^x$$

Quotient  $a^n / a^m = a^{n-m}$

$$a^n / b^n = (a/b)^n$$

Power  $(b^n)^m = b^{n*m}$

$$b^{n^m} = b^{(n^m)}$$

Negative  $b^{-n} = 1/b^n$

Derivative  $(x^n)' = nx^{n-1}$

$$(b^x)' = \ln(b) * b^x$$

Integral  $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Definition:  $\ln(x) = \int_1^x 1/t \, dt$

Observations:

$$\ln(1)=0$$

$$\ln(x)<0 \text{ if } 0<x<1$$

$$\ln(x)>0 \text{ if } 1<x$$

$x<0$ , doesn't work

$$\ln(xy) = \int_1^{xy} 1/t \, dt$$

$$\ln(x) + \ln(xy) = \int_1^x 1/t \, dt + \int_1^{xy} 1/t \, dt$$

We let the first term be  $\ln(x)$  and use u substitution for the second term and let  $u=t/x$

$$\text{We get } \ln(xy) = \ln(x) + \ln(y)$$

Definition:  $e^y$  is the inverse function of  $\ln(x)$

$$\ln(x)=y$$

$$x=e^y$$

Properties of  $e^y$

$$e^0=1$$

$$\ln(e)=1$$

$$e=e^1$$

$$\ln(e^y)=y$$

$$1/e^y * (e^y)' = 1; (e^y)' = e^y$$

From before, we know  $\ln(xy) = \ln(x) + \ln(y)$

We want to show that  $e^{x+y} = e^x e^y$

$$\ln(e^x) + \ln(e^y) = \ln(e^x e^y)$$

$$x + y = \ln(e^{x+y})$$