Math 3210-4/ Honors Foundations of Analysis/Fall 2016
Second Midterm (Due Wed November 2)

1. Prove the Heine-Borel Theorem (closed and bounded subsets are compact) for subsets of $\mathbb{R}^2$ in your own words and be sure to include a picture at a critical stage of the proof.

2. Suppose $\{s_n\}, \{t_n\}$ and $\{u_n\}$ are three sequences of complex numbers that converge to complex numbers s, t and u respectively. Show directly from the definition of convergence (and some arithmetic manipulations, as in the proof of Theorem 3.3) that:

(a) $\lim_{n\to\infty}(s_n + t_n + u_n) = s + t + u$

(b) $\lim_{n\to\infty}(s_n \cdot t_n \cdot u_n) = s \cdot t \cdot u$

3. Determine the lim sup and the lim inf of the following sequences. Explain your answer to each one!

(a) $s_n = e^n$

(b) $s_n = \sin(n)$ (in degrees).

(c) $s_n = \sin(n)$ (in radians).

4. Prove that the series:

$$\sum_{n=2}^{\infty} \frac{1}{(\log(n))^p}$$

diverges for all powers p.

5. List all the tests for convergence and divergence of series that we have covered with a sketch of the proof of each.

6. Find a power series with radius of convergence $R = 1$ and which fails to converges at exactly **two** points of the unit circle. Can you find a power series that fails to converge at p points for any p?

7. Reorder the alternating harmonic series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

to obtain a series that converges to 0.