Math 2200-002/Discrete Mathematics

Sample Problems

I. Logic.

A. Convert the following (true) statements into logical propositions.

- There is no largest natural number.
- Every subset of the natural numbers other than the empty set has a smallest element.
- Every real number has an additive inverse.
- Some real number has no multiplicative inverse.
- There are infinitely many prime numbers.
- $\sqrt{2}$ is not a rational number.
- Every natural number greater than 1 is the product of finitely many prime numbers.
- Every prime greater than 2 is an odd number.

B. Negate the following using DeMorgan's Laws:

- $p \wedge \neg q$
- $p \to q$
- $p \leftrightarrow q$
- $(\forall x)(\exists y)(P(x) \to Q(y))$.
- $(\exists x)(\forall y)(P(x) \to Q(y))$

II. Definitions. Using logical propositions, carefully define:

- The intersection and union of sets A and B (in a universe U).
- The power set of a set A.
- The complement of a set A.
- Injectivity, surjectivity and bijectivity of functions.
- A sequence of integers.
- Arithmetic and geometric sequences of real numbers.
- A recurrence relation for a sequence.
- The Fibonacci and Lucas sequences.
- A composite number.
- The division algorithm.
- The operations mod and div.
- Congruence mod m.
- Addition and multiplication mod m.
- The gcd and lcm of natural numbers m and n.

III. Short Answers. Carefully describe the following, using complete sentences and logical propositions.

- The Cartesian product of A and B.
- The graph of a function $f : A \to B$.

- Polynomial and exponential growth of sequences.
- The Euclidean algorithm for finding the gcd.
- Bézout's Theorem

IV. Proofs. Write down complete proofs of the following, carefully explaining every step of the proof.

- If $|S| = n$, then $|\mathcal{P}(S)| = 2^n$.
- $\sqrt{2}$ is not a rational number.
- There are infinitely many prime numbers.
- $1 + 3 + 5 + \cdots + (2n - 1) = n^2$
- The nth Fibonacci number is $(\phi^n + \psi^n)/\sqrt{5}$, where ϕ and ψ are the two solutions to the equation $x^2 = x + 1$.
- If m and n are relatively prime and d is any integer, then there are integers a and b so that: $am + bn = d$.
- $(p \to q) \leftrightarrow (\neg p \vee q) \leftrightarrow (\neg q \to \neg p)$

V. Some Problems.

- Find $\mathcal{P}(\{\emptyset, 0\})$.
- Show that for any two sets A and B, $\overline{A - B} = \overline{A} \cup B$.
- Solve the equation $13a + 43b = 1$.