1. (10 points) Newton’s Law of Gravitation says that the magnitude \( F \) of the force exerted by a body of mass \( m \) on a body of mass \( M \) is:

\[
F = \frac{GmM}{r^2}
\]

where \( G \) is the gravitational constant and \( r \) is the distance between the two bodies.

(a) Find \( F'(r) \) and explain its meaning.

(b) Suppose the Earth attracts an object with a force that decreases at a rate of 4 N/km when \( r = 10,000 \) km. How would the force of attraction to the same object be changing with \( r \) when \( r = 20,000 \) km? (Note: the numbers are not the same as in the homework problem!)

\[
F'(r) = \frac{-2GmM}{r^3}
\]

\( F'(r) \) is inversely proportional to \( r^3 \), decreasing with \( r \).

\[
-4 \text{ N/km} = \frac{-2GmM}{(10,000)^3} \Rightarrow GmM = 2 \cdot (10,000)^3
\]

\[
F'(20,000) = \frac{-2 \cdot (2 \cdot 10,000^3)}{(20,000)^3} = -\frac{2 \cdot 2}{8} = \frac{-\sqrt{2}}{4}
\]