Name:________________________________

Math 1310-004/Bertram
First Midterm Examination
October 6, 2014

Please indicate your reasoning and show all your work. You may use graphing calculators, but an unsupported answer will be marked wrong.

Relax and good luck!

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20 points) On the coordinates provided, sketch the graph of a single function $f(x)$ whose domain is all real numbers, and which satisfies all of the properties below:

(a) $\lim_{x\to-\infty} f(x) = -\infty$

(b) $f'(-2) = 0$ and $f''(-2) > 0$

(c) $\lim_{x\to 0} f(x) = 0$ but $f(x)$ is not continuous at 0.

(d) $f'(2) > 0$ and $f''(2) < 0$

(e) $\lim_{x\to\infty} f(x) = 0$

2. (20 points) Using the following table:

x	$f(x)$
0.9	0.73
0.99	0.97
1	1
1.01	1.03
1.1	1.33

(a) (10 points) Estimate the value of $f'(0)$.

(b) (10 points) Estimate the value of $f''(0)$.

3. Compute the following limits.

(a) (5 points)

$$\lim_{x \to 0} \frac{5x^3 - 7x^2}{x^3 + 2x^2} =$$

(b) (5 points)

$$\lim_{x \to \infty} \frac{5x^3 - 7x^2}{x^3 + 2x^2} =$$

(c) (5 points)

$$\lim_{x \to 0} \frac{\sin(2x)}{x} =$$

(d) (5 points)

$$\lim_{h \to 0} \frac{2^{2+h} - 4}{h} =$$

4. Compute the derivatives **and** second derivatives of the following:

(a) (10 points)

$$f(x) = \sqrt{1 - x^2}$$

$f'(x) =$

$f''(x) =$

(b) (10 points)

$$f(x) = e^x \tan(x)$$

$f'(x) =$

$f''(x) =$

5. (20 points) Sketch the graph of the function:

$$f(x) = \frac{x}{1+x^2}$$

indicating the domain and range and all critical points and inflection points (where the second derivative vanishes), and limits at $\pm\infty$. I am aware that you can just look at this on your calculator, so be sure to justify your graph.

Extra Credit. (10 points) Given functions $u = g(x)$ and $F(x) = f(u)$, prove the chain rule:

$$F'(x) = f'(u) \cdot g'(x)$$

Math 1310-004
The Cheat Sheet

Rules for Differentiating Combinations of Functions.

$$(cf)' = cf'$$
$$(f+g)' = f' + g', \quad (f-g)' = f' - g'$$
$$(fg)' = f'g + fg', \quad (1/g)' = -\frac{g'}{g^2}, \quad (f/g)' = \frac{f'g - fg'}{g^2}$$

Let $u = g(x)$ and $F(x) = f(g(x)) = f(u)$. Then $F'(x) = f'(u) \cdot g'(x)$.

The Basic Derivatives (so far).

$$\frac{d}{dx}(c) = 0, \quad \frac{d}{dx}(x^n) = nx^{n-1} \text{ as long as } n \neq 0$$
$$\frac{d}{dx}(e^x) = e^x, \quad \frac{d}{dx}(c^x) = c^x \ln(c) \text{ for all } c > 0$$
$$\frac{d}{dx}(\sin(x)) = \cos(x), \quad \frac{d}{dx}(\cos(x)) = -\sin(x)$$
$$\frac{d}{dx}(\tan(x)) = \sec^2(x), \quad \frac{d}{dx}(\cot(x)) = -\csc^2(x)$$
$$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x), \quad \frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$$

Some Trig Identities.

$$\sin^2(x) + \cos^2(x) = 1$$
$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

Exponent and Log Rules. Assume $b, c > 0$

$$c^{x+y} = c^x c^y, \quad \log_c(xy) = \log_c(x) + \log_c(y)$$
$$c^{-x} = 1/c^x, \quad \log_c(1/x) = -\log_c(x)$$
$$(c^x)^y = c^{xy}, \quad \log_c(x^y) = y\log_c(x)$$
$$b^x c^x = (bc)^x, \quad \log_b(c) = \ln(c)/\ln(b)$$

e is the number (approximate value: 2.718) satisfying:

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

and $\ln(x) = \log_e(x)$ is the "natural log."