Exercises

1. Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as $t$ increases.

1. $x = t^2 + t$, $y = t^2 - t$, $-2 < t < 2$
2. $x = t^2$, $y = t^3 - 4t$, $-3 < t < 3$
3. $x = \cos^2 t$, $y = 1 - \sin^2 t$, $0 \leq t \leq \frac{\pi}{2}$
4. $x = e^t + t$, $y = e^t - t$, $-2 < t < 2$

5. Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as $t$ increases.

(a) $x = 3t - 5$, $y = 2t + 1$
(b) $x = 1 + 3t$, $y = 2 - t^2$
(c) $x = 3t - 5$, $y = 2t + 1$
(d) $x = t^2$, $y = t^3$

9. Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as $t$ increases.

(a) $x = \sin t$, $y = \cos t$, $-\pi \leq t \leq \pi$
(b) $x = 2\sin t$, $y = 4 + \cos t$, $0 \leq t \leq \frac{\pi}{2}$
(c) $x = 5\sin t$, $y = 2\cos t$, $-\pi < t < \pi$
(d) $x = \sin t$, $y = \cos 2t$, $-\pi < t < \pi$

21. Suppose a curve is given by the parametric equations $x = f(t)$ and $y = g(t)$, where the range of $f$ is $[1, 4]$ and the range of $g$ is $[2, 3]$. What can you say about the curve?

22. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)—(d) with the parametric curves labeled I—IV. Give reasons for your choices.

23. Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch meiric curves $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as $t$ increases.

24. Describe the motion of a particle with position $(x, y)$ as $t$ varies in the given interval.

(a) $x = 3 + 2\cos t$, $y = 1 + 2\sin t$, $\frac{\pi}{2} < t < \frac{3\pi}{2}$
(b) $x = 2\sin t$, $y = 4 + \cos t$, $0 \leq t \leq \frac{\pi}{2}$
(c) $x = 5\sin t$, $y = 2\cos t$, $-\pi < t < \pi$
(d) $x = \sin t$, $y = \cos 2t$, $-\pi < t < \pi$

25. Consider a particle that moves in a closed path, making a complete oscillation whenever it swings back to its starting point. Such a particle is said to be a cycloid.

(a) Describe the motion of a particle that makes a complete oscillation whenever it swings back to its starting point.
(b) Sketch this curve and indicate with an arrow the direction in which the curve is traced in the given interval.

The Dutch physicist Huygens had already shown that the cycloid is the solution to the brachistochrone problem; that is, no matter where a particle is placed on an inverted cycloid, it will take the same time to slide to the bottom (see Figure 15). Huygens proposed that pendulum clocks (which he invented) should swing in cycloidal arcs because then the pendulum would take the same time to make a complete oscillation, regardless of the angle of swing. This is why pendulum clocks are said to be cycloidal clocks. The Dutch physicist Huygens had already shown that the cycloid is the solution to the brachistochrone problem; that is, no matter where a particle is placed on an inverted cycloid, it will take the same time to slide to the bottom (see Figure 15). Huygens proposed that pendulum clocks (which he invented) should swing in cycloidal arcs because then the pendulum would take the same time to make a complete oscillation, regardless of the angle of swing.

26. Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as $t$ increases.

(a) $x = \sin t$, $y = \cos 2t$
(b) $x = \ln t$, $y = t^2 - 1$, $1 < t < 2$

Graphing calculator or computer with graphing software required 1. Homework Hints available in TEC.