FIGURE 15

FIGURE 16

Later this curve arose in connection with the **brachistochrone problem**: Find the curve along which a particle will slide in the shortest time (under the influence of gravity) from a point A to a lower point B not directly beneath A. The Swiss mathematician John Bernoulli, who posed this problem in 1696, showed that among all possible curves that join A to B, as in Figure 15, the particle will take the least time sliding from A to B if the curve is part of an inverted arch of a cycloid.

The Dutch physicist Huygens had already shown that the cycloid is also the solution to the **tautochrone problem**; that is, no matter where a particle P is placed on an inverted cycloid, it takes the same time to slide to the bottom (see Figure 16). Huygens proposed that pendulum clocks (which he invented) should swing in cycloidal arcs because then the pendulum would take the same time to make a complete oscillation whether it swings through a wide or a small arc.

1.7 Exercises

1–4 Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

1. $x = t^2 + t, \quad y = t^2 - t, \quad -2 \leqslant t \leqslant 2$

2. $x = t^2, \quad y = t^3 - 4t, \quad -3 \leqslant t \leqslant 3$

3. $x = \cos^2 t, \quad y = 1 - \sin t, \quad 0 \leqslant t \leqslant \pi/2$

4. $x = e^{-t} + t, \quad y = e^t - t, \quad -2 \leqslant t \leqslant 2$

5–8

(a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

(b) Eliminate the parameter to find a Cartesian equation of the curve.

5. $x = 3t - 5, \quad y = 2t + 1$

6. $x = 1 + 3t, \quad y = 2 - t^2$

7. $x = \sqrt{t}, \quad y = 1 - t$

8. $x = t^2, \quad y = t^3$

9–16

(a) Eliminate the parameter to find a Cartesian equation of the curve.

(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

9. $x = \sin\frac{1}{2}\theta, \quad y = \cos\frac{1}{2}\theta, \quad -\pi \leqslant \theta \leqslant \pi$

10. $x = \frac{1}{2}\cos\theta, \quad y = 2\sin\theta, \quad 0 \leqslant \theta \leqslant \pi$

11. $x = \sin t, \quad y = \csc t, \quad 0 < t < \pi/2$

12. $x = \tan^2\theta, \quad y = \sec\theta, \quad -\pi/2 < \theta < \pi/2$

13. $x = e^{2t}, \quad y = t + 1$

14. $x = e^t - 1, \quad y = e^{2t}$

15. $x = \sin\theta, \quad y = \cos 2\theta$

16. $x = \ln t, \quad y = \sqrt{t}, \quad t \geqslant 1$

17–20 Describe the motion of a particle with position (x, y) as t varies in the given interval.

17. $x = 3 + 2\cos t, \quad y = 1 + 2\sin t, \quad \pi/2 \leqslant t \leqslant 3\pi/2$

18. $x = 2\sin t, \quad y = 4 + \cos t, \quad 0 \leqslant t \leqslant 3\pi/2$

19. $x = 5\sin t, \quad y = 2\cos t, \quad -\pi \leqslant t \leqslant 5\pi$

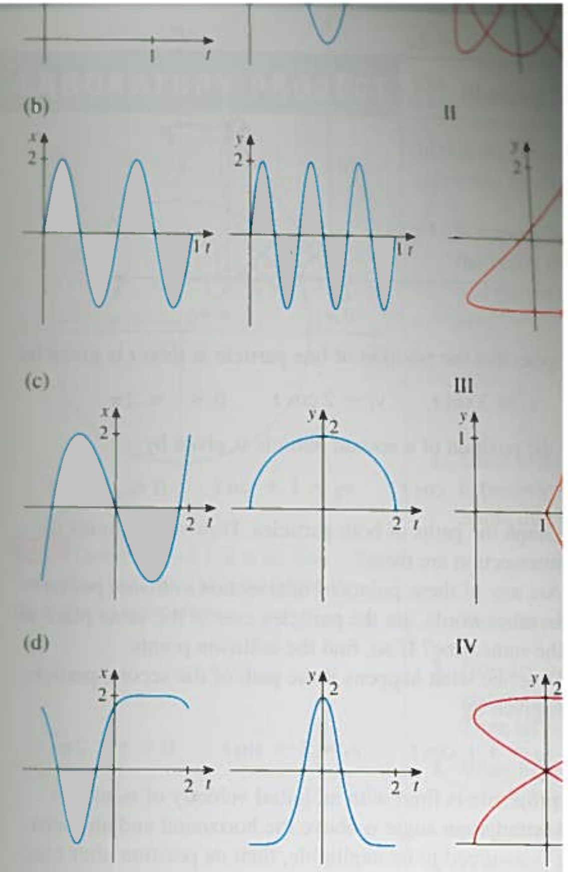
20. $x = \sin t, \quad y = \cos^2 t, \quad -2\pi \leqslant t \leqslant 2\pi$

21. Suppose a curve is given by the parametric equations $x = f(t)$, $y = g(t)$, where the range of f is $[1, 4]$ and the range of g is $[2, 3]$. What can you say about the curve?

22. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)–(d) with the parametric curves labeled I–IV. Give reasons for your choices.

Graphing calculator or computer with graphing software required

1. Homework Hints available in TEC



23–25 Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as t increases.

23.

24.