Math 1040
Midterm Examination
March 22, 2016

Relax and good luck!

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
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<tr>
<td>2</td>
<td>20</td>
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<td>3</td>
<td>20</td>
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<td>4</td>
<td>20</td>
<td></td>
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<tr>
<td>5</td>
<td>20</td>
<td></td>
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<tr>
<td>Total</td>
<td>100</td>
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</table>
1. Consider the following speeds (in mph) of 20 cars on the freeway:

<table>
<thead>
<tr>
<th>45</th>
<th>50</th>
<th>52</th>
<th>54</th>
<th>55</th>
<th>57</th>
<th>60</th>
<th>62</th>
<th>63</th>
<th>63</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>65</td>
<td>65</td>
<td>65</td>
<td>65</td>
<td>68</td>
<td>69</td>
<td>71</td>
<td>75</td>
<td>80</td>
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</tbody>
</table>

(a) (10 points) Find the three quartiles for the speeds:

\[
Q_1 = 55.0, \quad Q_2 = 60.3, \quad Q_3 = 66.5
\]

(b) (5 points) Draw a box-and-whisker plot for the data:

(c) (5 points) What percentile corresponds to the speed limit of 70 mph?

\[
\begin{align*}
\# \text{ of speeds below } 70 &= 17 \\
\# \text{ at speed } 70 &= 20 \\
70 \text{ mph Percentile} &= \frac{17}{20} = 85.0\%
\end{align*}
\]
2. The table below gives means and standard deviations for the heights of populations of men and women:

<table>
<thead>
<tr>
<th></th>
<th>Men's Heights</th>
<th>Women's Heights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>69.9 in</td>
<td>64.3 in</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>3.0 in</td>
<td>2.6 in</td>
</tr>
</tbody>
</table>

(a) (5 points) Find the z-score for a 5' (= 60 inches) tall man.

\[ z = \frac{x - \mu}{\sigma} = \frac{60 - 69.9}{3} = -3.3 \]

(b) (5 points) Find the z-score for a 5' (= 60 inches) tall woman.

\[ z = \frac{x - \mu}{\sigma} = \frac{60 - 64.3}{2.6} \approx -1.65 \]

(c) (5 points) How tall (or short) must a man be to be considered unusual? (Recall that unusual is a z-score of more than 2 or less than -2).

\[ z > 2 \Rightarrow \frac{x - 69.9}{3} > 2 \Rightarrow x > 69.9 + 6 = 75.9 \text{ in} \sim 6' 4" \]

\[ z < -2 \Rightarrow \frac{x - 69.9}{3} < -2 \Rightarrow x < 69.9 - 6 = 63.9 \text{ in} \sim 5' 4" \]

(d) (5 points) How tall (or short) must a woman be to be considered unusual?

\[ z > 2 \Rightarrow \frac{x - 64.3}{2.6} > 2 \Rightarrow x > 64.3 + 5.2 = 69.5 \text{ in} \sim 5' 9 1/2" \]

\[ z < -2 \Rightarrow \frac{x - 64.3}{2.6} < -2 \Rightarrow x < 64.3 - 5.2 = 59.1 \text{ in} \sim 4' 11" \]
3. Assume that the probability that a child is a girl is 1/2. For a family with four children, what is the probability that:

(a) (5 points) All the children are girls?

\[ P(4 \text{ girls}) = P(\text{girl}) \times P(\text{girl}) \times P(\text{girl}) \times P(\text{girl}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16} \]

(b) (5 points) One or more of the children is a girl?

\[ P(\geq 1 \text{ girl}) = 1 - P(4 \text{ boys}) = 1 - \frac{1}{16} = \frac{15}{16} \]

(c) (5 points) Exactly one of the children is a girl?

\[ \text{Tree of possibilities} \]

(d) (5 points) Exactly two of the children are girls?

\[ \text{Tree of possibilities} \]
4. The following table is the result of a survey of a total of 100 men and women asking them whether they were smokers or non-smokers:

<table>
<thead>
<tr>
<th></th>
<th>Non-Smoker</th>
<th>Smoker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Female</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) (3 points each) Find all the following empirical probabilities:

\[ P(\text{Smoker}) = \frac{25}{100} = 0.25 \]

\[ P(\text{Male}) = \frac{60}{100} = 0.60 \]

\[ P(\text{Male and Smoker}) = \frac{15}{100} = 0.15 \]

\[ P(\text{Male or Smoker}) = \frac{70}{100} = 0.70 \]

\[ P(\text{Smoker|Male}) = \frac{15}{60} = 0.25 \]

(b) (5 points) According to the table, is being a smoker independent of being male? Explain your answer.

Since \[ P(\text{Smoker}) = P(\text{Smoker|Male}) \]

it follows that being a smoker is independent of being male.
5. A scholarship committee has 3 identical awards to give to top students. They are considering 15 applicants, 5 of whom are majoring in mathematics.

(a) (5 points) In how many different ways can they make the awards?

\[
\binom{15}{3} = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} = 455
\]

(b) (5 points) In how many different ways can they make the awards, so that none of them go to math majors?

\[
\binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120
\]

(c) (5 points) In how many different ways can they make them so that exactly one of them go to a math major?

\[
\binom{10}{2} \times \binom{5}{1} = \frac{10 \cdot 9}{2 \cdot 1} \times 5 = 225
\]

(d) (5 points) If the awards are made at random, what is the probability that one or more of them go to a math major?

\[
1 - P(\text{none to a math major})
\]

\[
= 1 - \frac{120}{455} \approx 0.74
\]