Quiz no. 5 (1220-5 Calculus II, Fall 2006)
November 7, 2006

25 min. No symbolic calculators allowed (TI-89 and similar)!
(TI-86 or lower are allowed.)

1. (4 points) Determine the value of the following series:
\[
\frac{1}{2} + \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \cdots =
\]

2. (12 points) Find out which of the following series converges. Indicate which test you use.

(a) \[
\sum_{n=2}^{\infty} \frac{n}{e^n}
\]

(b) \[
\sum_{n=2}^{\infty} \frac{\sqrt{n + 1}}{\sqrt{n^3 + 1}}
\]
(c)

\[ \sum_{k=1}^{\infty} \frac{n^2}{n!} \]

3. (5 points) Compute this series: Hint: Use a partial fraction decomposition.

\[ \sum_{k=1}^{\infty} \frac{1}{(2k-1)(2k+1)} = \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \ldots \]

4. (4 points) Assume you know that the sequence given by

\[ a_1 = 2, \quad a_{n+1} = a_n + \frac{1}{a_n} - 1 \]

has a limit \( L = \lim_{n \to \infty} a_n \). Determine the limit by finding an equation for \( L \).