1. (8 points) Compute the value of the following series:

(a) \[\frac{1}{2} + \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \frac{16}{243} + \cdots =\]

(b) *Hint: Find a formula for \(S_n\).*

\[\sum_{k=2}^{\infty} \frac{1}{k^2} - \frac{1}{(k+1)^2} =\]

2. (6 points) A square with side length of 1 is divided into 4 smaller squares of side length \(\frac{1}{2}\), and the lower right square is shaded. Then this is repeated with the upper left square, and so on, as in figure 1. Find the total area of the shaded region.

![Figure 1: Find the total area of the shaded region.](image)
3. (8 points) Which of the following two series converges? Indicate which test you use.

(a) \( \sum_{n=1}^{\infty} \frac{n^2 + n + 1}{3^n} = \)

(b) \( \sum_{n=1}^{\infty} \frac{\ln n}{n^2} = \)

4. (6 points) Does the following series converge? Does it converge absolutely?

\( \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n + 1}}{\sqrt{n^3 + n^2}} = \)

5. (8 points) For each of the following sequences, determine whether it converges, and compute the value of the limit if it does.

(a) \( a_n = \frac{n^3}{4^n} \)
(b) **Hint:** Use the squeeze theorem. $a_n = \frac{\sin n}{n \sqrt{n}}$

6. (4 points) The sequence $(a_n)$ is defined by $a_1 = 1, a_{n+1} = \frac{2}{a_n} + 1$. It can be shown that this sequence converges. Determine its limit.

7. (6 points) Find the convergence set of the following power series:

   $p(x) = \sum_{n=0}^{\infty} \frac{(x + 3)^n}{n^2}$

8. (8 points) Find the taylor series of $f(x) = \cosh x$ by computing its derivatives. Give a formula for the $n$-th term of the power series, and determine its convergence set.
9. (10 points) Find a power series for each of the following functions. You can use the formula for the geometric series, and any of the power series listed at the bottom of the page.

(a) \[ \frac{x}{1 + x^7} = \]

(b) \[ (1 + x) \ln(1 + x) = \]

(c) \[ \int_{0}^{x} e^{t^4} \, dt = \]

Power series you can use in problem no. 9:

\[ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \]

\[ \ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} \]