Your name: ____________________

Midterm no. 3 (1220-5 Calculus II, Fall 2006)

November 21, 2006

No symbolic calculators allowed (TI-89 and similar)! (TI-86 or lower are allowed.) **60 min.**

1. (8 points) Compute the **value** of the folloing series:

   (a)

   $$\frac{1}{2}+\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\frac{8}{81}+\frac{16}{243}+\cdots=$$

   (b) *Hint: Find a formula for* S_n.

   $$\sum_{k=2}^{\infty}\frac{1}{k^2}-\frac{1}{(k+1)^2}=$$

2. (6 points) A square with side length of 1 is divided into 4 smaller squares of side length $\frac{1}{2}$, and the lower right square is shaded. Then this is repeated with the upper left square, and so on, as in figure 1. Find the total area of the shaded region.

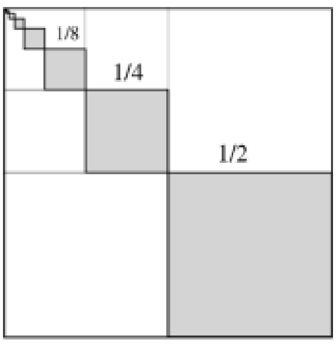

Figure 1: Find the total area of the shades region.

3. (8 points) Which of the following two series converges? Indicate which test you use.

   (a)

$$\sum_{n=1}^{\infty} \frac{n^2+n+1}{3^n} =$$

   (b)

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^2} =$$

4. (6 points) Does the following series converge? Does it converge absolutely?

$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n+1}}{\sqrt{n^3+n^2}} =$$

5. (8 points) For each of the following **sequences**, determine whether it converges, and compute the value of the limit if it does.

   (a) $a_n = \frac{n^3}{3^n}$

(b) *Hint: Use the the squeeze theorem.* $a_n = \frac{\sin n}{n \cdot \sqrt{n}}$

6. (4 points) The sequence (a_n) is defined by $a_1 = 1, a_{n+1} = \frac{2}{a_n} + 1$. It can be shown that this sequence converges. Determine its limit.

7. (6 points) Find the convergence set of the following power series:

$$p(x) = \sum_{n=0}^{\infty} \frac{(x+3)^n}{n^2}$$

8. (8 points) Find the taylor series of $f(x) = \cosh x$ by computing its derivatives. Give a formula for the n-th term of the power series, and determine its convergence set.

Please turn page

9. (10 points) Find a power series for each of the following functions. You can use the formula for the geometric series, and any of the power series listed at the bottom of the page.

(a)

$$\frac{x}{1+x^7} =$$

(b)

$$(1+x)\ln(1+x) =$$

(c)

$$\int_0^x e^{t^4}\,dt =$$

Power series you can use in problem no. 9:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{n} \frac{x^n}{n!}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots = \sum_{n=1}^{n} (-1)^{n+1} \frac{x^n}{n}$$