

Your name: \_\_\_\_\_

Midterm no. 1 (1220-5 Calculus II, Fall 2006)  
October 24, 2006

**No symbolic calculators allowed (TI-89 and similar)! (TI-86 or lower are allowed.) 60 min.**

1. (4 points)

$$\int \frac{x \sin x^2}{\cos x^2} dx =$$

2. (4 points)

$$\int x \ln x dx =$$

3. (6 points) *Hint: You can't use partial fraction decomposition directly.*

$$\int \frac{x^3 + x^2 - x}{x^2 - 1} dx =$$

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4. (6 points)

$$\int_0^{\sqrt{3}} \frac{x^2 + 2x + 1}{(x^2 + 1)^2} dx =$$

5. (6 points) *Hint: Apply partial integration twice. Consider the equation you get.*

$$\int \sin x \sinh x dx =$$

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6. (12 points) Evaluate each of the following improper integrals, or show that it diverges:

(a)

$$\int_0^1 \frac{1}{(x-1)^3} dx =$$

(b) *Hint: Use completion of the square.*

$$\int_0^1 \frac{1}{\sqrt{x^2 + 2x}} dx =$$

7. (8 points) Find the area of the shape that is bordered by  $y = \frac{1}{x\sqrt{2x-1}}$ ,  $y = 0$  and  $x = 1$  and extends to infinity to the right. *Hint: Use a rationalizing substitution.*

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8. (12 points) Find the following limits:

(a) *You may use that*  $\lim_{x \rightarrow +\infty} \sinh x = +\infty$  *and*  $\lim_{x \rightarrow +\infty} \cosh x = +\infty$ .

$$\lim_{x \rightarrow +\infty} \frac{x^2}{\sinh x} =$$

(b)

$$\lim_{x \rightarrow 0^+} (\cos x)^{\frac{1}{x^2}} =$$

(c)

$$\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{\sin x} \right) =$$