Midterm no. 1 (1220-5 Calculus II, Fall 2006)

Your name: _____

September 26, 2006

No symbolic calculators allowed (TI-89 and similar)! (TI-86 or lower are allowed.)
   60 min.

1. *(10 points)* Compute the derivatives $D_x y$ for the following functions.

   (a) $y = x \ln x - x$

   (b) $y = \tan^{-1}(x^2)$

   (c) $y = x^{\sin x}$

2. *(10 points)* Compute the following integrals.

   (a)
   $$\int x^2 e^{1+x^3} \, dx =$$

(b)

$$\int \frac{1+x}{\sqrt{x-1}}dx =$$

3. *(6 points)* Find the area between the x-axis and $y = x\sqrt{1-x^2}$ for $0 \le x \le 1$.

4. *(6 points)* Show that the function $f(x) = x^6 + x^4 + 1$ has an inverse function $f^{-1}(y)$ for $x > 0$. Find the derivative $(f^{-1})'(3)$.

Hint: You may guess x such that $f(x) = 3$. ***Don't*** *try to find a formula for $f^{-1}(y)$.*

5. *(6 points)* Find the solution of the differential equation

$$y' + y\frac{\sin x}{\cos x} = \frac{1}{\cos x}$$

such that $y = 1$ for $x = 0$.

You may assume $-\frac{\pi}{2} < x < \frac{\pi}{2}$, so that $\cos x > 0$.

6. *(8 points)*

   (a) Give the definition of $\ln x$. What is $D_x \ln x$? Use this to show that $\ln 3x = \ln x + \ln 3$ for all $x > 0$.

   (b) Show that $(\cosh x)^2 + (\sinh x)^2 = \cosh 2x$.

7. *(4 points)* Plutonium-239 is decaying exponentially and has a half-life time of 24110 years. How much will be left of 100 g after 10 years? Give the differential equation for the amount of Plutonium over time.

8. *(4 points)* A nuclear reactor is producing $10g$ of Plutonium-239 per year; at the same time, the Plutonium is decaying (see previous problem). Give the differential equation for the amount $P(t)$ of Plutonium over time.

   You don't need to solve the differential equation!

9. *(6 points)* A car is driving along I-80 through the Valley eastwards at a speed of 60 mph, directly heading towards Mount Olympus. The peak of Mount Olympus is 0.9 miles above the level of the Valley.

   When the car is 5 miles West of Mount Olympus, what is the angle of elevation of the line of sight to the peak? What is the rate of change of this angle? (Give the answers in radian and radian per minute.)