Math 6720 *Spring 2011*

Applied Complex Variables and Asymptotic Methods

Take Home MidTerm **March 3, 2011 - due March 8, 2011.**

1. Let $f(z)$ be analytic at z_0. Prove that there exists a point z_1 such that $|f(z_1)| > |f(z_0)|$ unless $f(z)$ is constant.

   Does there exist a point z_2 such that $|f(z_2)| < |f(z_0)|$?

2. Consdider the Taylor series expansion of the function

$$\frac{1}{\cosh x} \qquad (x \text{ is a real variable})$$

   around the origin ($x_0 = 0$). Find the radius of convergence of this series.

3. Let the functions $u_1(x, y)$ and $u_2(x, y)$ be harmonic in a bounded domain D of the real (x, y)-plane, and continuous in $\bar{D}$. Prove that if these functions coinside on ∂D, then they are identically equal in D.

4. Solve the Laplace equation $\Delta\phi = 0$ subject to the boundary conditions:

$$\phi = a \text{ on } |z| = 1$$
$$\phi = b \text{ on } |z - 1| = 2$$

   ($z = x + iy$, a and b are real numbers). Check that your solution indeed satisfies the boundary conditions.

5. Evaluate the integral

$$I = \int_{-\infty}^{+\infty} \frac{\cos x - \cos a}{x^2 - a^2} dx \qquad (a \text{ is a positive parameter}).$$

6. Evaluate the integral

$$I = \int_0^{\infty} \frac{dx}{x^{100} + 1} \, .$$