1. Find the general solution $u(x,t)$ of the equation

$$u_{xtt} + u_x = 0.$$ 

2. Function $u(x,t)$ satisfies the following equation

$$u_t - x^2 u_x = 0.$$ 

Is it true that $u(1,1) = u(2,2)$? Is it true that $u(1,1) = u(2,\frac{1}{2})$? Explain.

3. Consider diffusion of some perfume on the whole axis $x$. The concentration of the perfume $u(x,t)$ satisfies the diffusion equation $u_t = u_{xx}$. The center of mass of the perfume cloud has coordinate

$$Q(t) = \int_{-\infty}^{+\infty} xu(x,t)dx.$$ 

Show: The diffusion equation implies that the center of mass doesn’t move, $\dot{Q} = 0$, (in accordance with physical considerations).

4. Find the general solution of the PDE

$$u_{xx} + 3u_{xy} + 2u_{yy} = 0.$$ 

5. Consider the wave equation $u_{tt} = c^2 u_{xx}$ on the whole line $-\infty < x < +\infty$. The initial condition

$$u(x,0) = \begin{cases} 
    x & \text{if } 0 < x < 1, \\
    2 - x & \text{if } 1 < x < 2, \\
    x - 3 & \text{if } 3 < x < 5, \\
    7 - x & \text{if } 5 < x < 7, \\
    0 & \text{otherwise,}
\end{cases}$$

$u_t(x,0) = 0$. 

What is the maximum value of the signal $u$? At which point $x$ and at which instant $t$ does it take place?

6. Consider the boundary value problem for the function $u(x,t)$ in the semi-strip $0 \leq x \leq 1$, $t \geq 0$:

$$u_{xt} = 0, \quad u(0,t) = 0, \quad u(1,t) = \sin t.$$ 

Show that the solution of this problem does not exist.