

Math 3160

Applied Complex Variables

Spring 2008

NAME:.....

MidTerm3

April 17, 2008.

1. Evaluate the following integral (using residues)

$$\int_{-\infty}^{+\infty} \frac{\sin x \, dx}{x^2 + 4x + 5}.$$

2. Evaluate the following integral (using residues)

$$\int_0^{2\pi} \frac{d\theta}{1 + a \cos \theta}$$

(where a is a parameter, $-1 < a < 1$)

3. Find the image of the semi-infinite strip $x > 0, 0 < y < 2$ under the transformation $w = iz + 1$ ($z = x + iy, w = u + iv$). Sketch the strip and its image.

4. Find the image of the infinite strip $0 < y < 1/(2c)$ under the transformation $w = 1/z$ ($z = x + iy$, $w = u + iv$). Sketch the strip and its image.

5. Find the linear fractional transformation that maps points $z_1 = \infty$, $z_2 = i$, $z_3 = 0$ into points $w_1 = 0$, $w_2 = i$, $w_3 = \infty$ (respectively). Into what curve the line $x = 1$ is transformed? Sketch the line and its image ($z = x + iy$, $w = u + iv$).