

Math 3160

Applied Complex Variables

Spring 2008

NAME:.....

MidTerm2

March 6, 2008.

1. Write the Maclaurin series representation of the function $f(z) = \frac{1}{1-z^3}$, and point out how it follows that

$$f^{(3n+1)}(0) = 0 \quad \text{and} \quad f^{(3n+2)}(0) = 0 \quad (n = 0, 1, 2, \dots).$$

2. Give two Laurent series expansions *in powers of z* for the function

$$f(z) = \frac{1}{z^2(i-z)},$$

and specify the regions in which those expansions are valid.

3. Find the residue at $z = 0$ of the function

(a) $f(z) = \frac{\cos z}{z}$,

(b) $f(z) = z \cos\left(\frac{1}{z}\right)$,

(c) $f(z) = \frac{1 - \cos z}{z^2}$.

4. Use residues to find the integral

$$\int_0^{\infty} \frac{x^2 dx}{(x^2 + 1)(x^2 + 4)}.$$

5. Use residues to find the integral

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}.$$