Math 3160 **Applied Complex Variables** **Spring 2008**

NAME:.....

MidTerm1 **February 21, 2008.**

1. (a) Sketch the set of points determined by the condition: $|z + i| \leq 3$.
   (b) Sketch the set of points determined by the condition: $\Im z > 1$ ($\Im$ means the imaginary part).

2. (a) What is the square root of $2i$?
   (b) What is $\log(2i)$?

3. Sketch the region onto which the sector

$$r \leq 1, \quad 0 \leq \Theta < \pi/4$$

is mapped by the transformation $w = z^2$. $[r = |z|, \quad \Theta = Arg(z).]$

4. Show that the function $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic, and find a harmonic conjugate $v(x, y)$.

5. Find all roots of the equation $\cos z = 2$.

6. Evaluate

$$\int_\Gamma f(z)dz,$$

where $f(z)$ is the branch

$$z^{-1+i} = e^{(-1+i)\log z} \qquad (|z| > 0, 0 < \arg z < 2\pi),$$

and Γ is the positively oriented circle $|z| = 1$.

7. Find the Maclaurin series expansion of the function $f(z) = \frac{z}{z^4+9}$.
   Where is it valid? (In other words: Where does it converge?)