HW6 **NAME: .....** **due Wednesday, 2/21/2018**

1. **Jordan's lemma** to neglect integrals

$$I(R) = \int_{C_R} f(z)\, e^{iaz}\, dz \qquad (a > 0) \quad \text{over "large" semi-circle } C_R:\ z = Re^{i\phi}\ \ (0 \le \phi \le \pi), \qquad R \to \infty.$$

Prove: If on C_R, $|f(z)| < F(R) \to 0$ [for some function $F(R)$], then $I(R) \to 0$.

2. The (single-valued) function $f(z) = \sqrt{z(z-1)(z-4)}$ is defined by
branch cuts $\quad z = -it, \quad z = 1 + it, \quad z = 4 - it \quad (0 \le t < \infty) \quad$ and value $\quad f(2) = -2i.$

Evaluate

(a) $f(-3)$,

(b) $f(1/2)$,

(c) $f(5)$.

3. The (single-valued) function

$$f(z) = \frac{(z-16)^{1/3}(z-20)^{1/2}}{z-1}$$

is defined by branch cuts $\quad z = 16 + it, \quad z = 20 - it \quad (0 \le t < \infty)$ and value $f(24) = \frac{4}{23}$.
Evaluate $\oint_C f(z)dz$ over the circle $C: \ |z-2| = 2$.

4. We are given multivalued analytic function

$$f(z) = [(z-1)(z-2)]^{1/3}.$$

What branch cuts would you make to separate single-valued branches?

5. **Describe singularities (their locations, types, and orders).**

(a) Describe singularities (their locations and types) for the function $f(z) = \frac{(z-1)^2 \cos^2(z/2)}{(z+1)^3 z(z-\pi)}$.

(b) Consider the multi-valued function $f(z) = \frac{\sqrt{z}-1}{z-1}$. To separate $f(z)$ into single-valued branches, make a brunch cut along the negative real semi-axis. What isolated singularities (their locations and types) do these branches have?

(c) Consider the multi-valued function $f(z) = (1+z)^{1/z}$. To separate $f(z)$ into single-valued branches, make a brunch cut $z = -t$ $(1 \leq t < \infty)$. What isolated singularities (their locations and types) do these branches have?