HW6 NAME: ................................................................. due Wednesday, 2/21/2018

1. **Jordan’s lemma** to neglect integrals

\[ I(R) = \int_{C_R} f(z) e^{iaz} \, dz \quad (a > 0) \text{ over “large” semi-circle } C_R : z = Re^{i\phi} \quad (0 \leq \phi \leq \pi), \quad R \to \infty. \]

**Prove:** If on \( C_R \), \( |f(z)| < F(R) \to 0 \) [for some function \( F(R) \)], then \( I(R) \to 0. \)
2. The (single-valued) function $f(z) = \sqrt{z(z-1)(z-4)}$ is defined by branch cuts $z = -it$, $z = 1 + it$, $z = 4 - it$ ($0 \leq t < \infty$) and value $f(2) = -2i$.

Evaluate

(a) $f(-3)$,
(b) $f(1/2)$,
(c) $f(5)$.
3. The (single-valued) function

\[ f(z) = \frac{(z - 16)^{1/3}(z - 20)^{1/2}}{z - 1} \]

is defined by branch cuts \( z = 16 + it, \quad z = 20 - it \quad (0 \leq t < \infty) \) and value \( f(24) = \frac{4}{23} \).

Evaluate \( \oint_C f(z)dz \) over the circle \( C : |z - 2| = 2. \)
4. We are given multivalued analytic function

\[ f(z) = [(z - 1)(z - 2)]^{1/3}. \]

What branch cuts would you make to separate single-valued branches?
5. Describe singularities (their locations, types, and orders).

(a) Describe singularities (their locations and types) for the function \( f(z) = \frac{(z-1)^2 \cos^2(z/2)}{(z+1)^5 z(z-\pi)} \).

(b) Consider the multi-valued function \( f(z) = \sqrt{\frac{z-1}{z+1}} \). To separate \( f(z) \) into single-valued branches, make a brunch cut along the negative real semi-axis. What isolated singularities (their locations and types) do these branches have?

(c) Consider the multi-valued function \( f(z) = (1 + z)^{1/z} \). To separate \( f(z) \) into single-valued branches, make a brunch cut \( z = -t \) (1 \( \leq t < \infty \)). What isolated singularities (their locations and types) do these branches have?